

Ways of Seeing Geometrical Meaning in Different Situations

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Abstract

This thesis set out to challenge the traditional approach to the study of geometrical understanding which has assumed that conceiving and interpreting shapes or forms is the result of logical and mental interaction between an individual and geometrical objects and that the production of geometrical meaning is motivated by the stimulus of the external structure of a visual text. By way of contrast, this study makes the case that geometrical meaning is socially and contextually produced.

The research has two interconnected strands. The first strand is theoretical aiming to develop a framework for the study of geometrical understanding drawing on concepts from Mikhail Bakhtin, Umberto Eco and Gunther Kress. The second is empirical aiming to collect data whose analysis will inform and be informed by this theoretical framework. For this study, three groups of people who differed radically in terms of their geometrical experiences, socio-economic and educational backgrounds were interviewed in order to examine their interpretations of geometrical elements exhibited in different settings.

The theoretical work of this thesis led to a framework for understanding geometry comprising 'sign', 'sign-functions', 'visual text', and 'heteroglossia'. Analysis of the data from empirical study in terms of this framework revealed the importance of the dynamics for visual experience as a process for communicating and of signifying, and how this relationship was itself dependent on the material conditions and contextual dynamics in which the meanings were constructed. The thesis concludes with an assessment of its potential contribution to redress the balance between learning about geometry and learning through geometry.

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1 Introduction to the problem: a study of geometrical understanding

1.1 The aim of the research

The aim of this research is to offer a re-vision of the study of geometrical understanding and to explore the implications of this alternative perspective. Traditionally, mathematics educational studies have adopted an approach which defines the learner as an isolated and autonomous subject, interpreting and understanding shapes and forms through the interaction of mind and geometrical objects. There has been a tendency to think geometrical meaning is derived from the stimulus material: the objects' structural features. Traditional methods thus propose two related assumptions: first, that a visual text can be interpreted individually and autonomously and, second, that a visual text defines meaning in its entirety (the formalist position). Challenging these premises forms the impetus and underlying theme of my research. In so doing, this study takes the position that a visual text - here geometrical material and pictorial figures - is polysemic, that is, it may be interpreted variously, according to different contexts and the observer's position within multiple social discourses.

This research has two converging strands. Initially, I will develop a theoretical framework for the study of geometrical understanding, drawing on concepts from Bakhtin, Eco and Kress. I shall then undertake an empirical study whose analysis will examine and demonstrate this framework.

The present chapter presents an overview of the problem under study and some key theoretical concepts of analysis. Two central schools of thought will be considered: first, the psychological orientation of much of the study of mathematics education will be analysed and its limitations discussed. Secondly, I shall delineate some connections between cognitive psychology and traditional semiology in order to offer some reflections on the intellectual context from which the trajectory of this work originated. Having next briefly described the

participants in the empirical study, the strategies adopted to obtain information and the material used, I shall illustrate the relationship between the theory and empirical data. The chapter concludes with a summary of the main terms of my theoretical position and the central questions of the research.

1.2 Background to the problem

Although a Piagetian perspective still remains the dominant approach in the field of mathematics education (including the study of geometrical thinking), in some areas of current research this model has been criticized. In particular, the universal status of mathematical knowledge and Piaget's excessive emphasis on abstraction has been questioned. Piaget supposes that knowledge of mathematics is essentially value-free and ahistorical. Those that challenge Piagetian notions introduce new possibilities for rethinking and redefining mathematical knowledge. They do so by beginning with the premise that mathematical knowledge is socially constructed.

The incorporation of sociocultural dimensions of mathematics education as a focus of analysis has encouraged researchers to pursue different routes. Lave (1988), for example, has argued that mathematical ideas are mediated by specific contexts. Thus, mathematical knowledge operates through a specific cultural activity. This means that school mathematics - as an academic institution with its own goals and means - cannot simply be transferred into another reality outside academic life. From another research perspective, the work of Nunes et al. (1993) investigated the kind of cognitive processes involved in formal and informal mathematics. The main questions these researchers addressed were: to what extent does the social situation influence human activities? What differences do social circumstances make? For instance, what are the differences between mathematics as a science taught by a teacher and mathematics as part of one's living activity? Another question raised by these authors refers to the widespread assumption that school failure could be

interpreted as cognitive or cultural deficit. What is clear from this work is that 'cognitive skills' should not be examined in 'the abstract' but in social contexts.

In a different way, this criticism of universalism and of an autonomous subject also played a crucial role in Walkerdine's work (see Walkerdine 1988). Walkerdine analysed the production of language as a 'discursive practice' within the daily life of young children. In order to address how language and cultural/ideological practices operated, Walkerdine drew upon Lacan's psychoanalysis, Saussure's linguistics and Foucault's semiotics, which allowed her to link the ways of learning mathematics to larger issues of social inequality.

All these studies have taken numbers and arithmetic as their mathematical core. When one reviews the territory covered by studies within the geometrical domain, however, the same shift in emphasis is not so noticeable. Indeed, an examination of the current literature concerning geometry suggests that most researchers, while not neglecting social and cultural influences, tend not to take these as their focal points of enquiry. Geometry education (theory and research) remains largely guided by a Piagetian theoretical approach, which privileges geometry as a cognitive construction, beginning at a perceptual level and continuing to a more representational one. In geometry education, the idea of perception/representation is used to conceptualise the relation between an individual's internal thought operations and external reality. Thus the interaction between inner and outer worlds is established through the processes of perception and of cognition.

Implicitly, this perspective argues that the production of meaning is motivated by stimuli from physical objects and mental processes. Rather than approach meaning - in this case geometrical meaning - from this point of view, I shall explore how meaning is produced by different social practices. In particular, how formal and non-formal schooling influence the 'voices' of the participants in different situations.

1.3 From the Psychological to the Semiological

At first sight, shifting the mode of reflection from the psychological to the semiological seemed to indicate to me an unambiguous journey since the guidelines of semiological concepts would demarcate the territories of individual and social construction of meaning. However, closer examination of the semiological perspective revealed that important connections needed to be made between cognitive psychology and traditional semiology, in so far as these two disciplines defined the question of the production of meaning. In investigating this connection, I shall not be concerned with identifying in detail the specific theoretical input of these two approaches. Rather I shall be concerned to point out some broad connections between the methodological positions of the logico-analytic approach in the field of geometry and those of traditional semiology. In doing this, my intention is to indicate the trajectory of the development of my theoretical ideas and to introduce the conceptual map underpinning the empirical research which will be developed in Chapter 2.

In order to consider how meaning is produced in these two disciplines, cognitive psychology and semiology, I shall examine how two fundamental psychological concepts, perception and representation, may be related to two fundamental semiological concepts, arbitrary and motivated sign.

According to Saussurean semiology's founding principle, meaning is independent of any external reality, having no direct reference to real things (physical form). Indeed, the relation between the signifier (the object) and the signified (the concept to which a signifier refers) - is conventional and unmotivated. Therefore the sign, generated by a symbiosis of signifier and signified, is arbitrary. This implies, as Hall (1986:34) points out:

That things and events in the real world do not contain or propose their own integral, single and intrinsic meaning, which is merely transferred through language. (...) This approach dethroned the referential notion of language (...) where the meaning of a particular term or sentence could be validated simply by looking at what, in the real world, it referenced.

From this perspective, elements of a language acquire meaning because they are inserted in a social system of relations in which the elements are ruled by laws of differences and oppositions.

Contrary to an arbitrary sign, there is a visual sign (iconic or motivated) where the relation between signifier and signified is not purely conventional, but one of resemblance or likeness. The iconic sign was studied by two semiologists, Peirce and Morris, who suggested that the relation between an image and its object is based on criteria of similarity, such as analogy and shared properties.

In semiotics, this perspective was sustained and reinforced by the idea of referent as proposed by Frege, a logician. The central point of the Fregean perspective is that the signified of a term (which may be visual or verbal) has a direct and unambiguous connection with the term or object to which it refers. In this perspective, the actual object to which the sign refers is the point of departure for analysing the meaning of images.

Let us now explore where the logico-analytical researchers situate meaning with respect to a visual text. Answers to this question might be sought in two directions: (i) in a direct reading of a physical form in which meaning (perceptual knowledge) is guaranteed by the objects' external structure - here, structure and meaning exist at the same level; (ii) in an analytical reading of a physical form in which the external structure of the object loses its importance to some degree since meaning (representation) is not totally dependent on physical structure but rather resides in concepts abstracted from it. Therefore, at this level, physical structure is unimportant except in so far as it suggests idealisations and abstractions.

This analysis suggests that iconic sign is connected with the question of perception, and more particularly, visual perception. From both perspectives, meaning is derived from the structural features of the objects, the stimulus material. Semiotically speaking, the signifier coincides with the signified. At the second level, the representational one, meaning does not refer to the physical

material as such but to the conception of that material, thus, the sign does not have a direct correspondence between the signifier and signified. This aspect might correspond to the model of analysis proposed by Saussure in which the central principle is that language "does not lie in the things themselves, but in the relationships we construct, and then perceive, between them" (Hawkes 1977:17). As mentioned earlier, within the Saussurean model, signs acquire meaning because they are inserted in a social system of relations whose elements are ruled by laws of differences and oppositions. These laws are incontestable social conventions and norms which the individual has no choice but to accept and adopt. Here, the inward structure of the sign is emphasised, separated from the cultural or social context in which it is produced. By this means, the distinction between *langue* and *parole* in Saussure's linguistic model was made following Cartesian dualism by means of a clear "separation between the actual speech act and the abstract system of norms internalised by the linguistic competence of the speakers". This implies separating "what is social from what is individual" and "what is essential from what is accessory and more or less accidental" (Bakhtin/Voloshinov 1973:163-4).

Similarly, geometry - as a form of language - is polarised by the logico-analytical approach, which distinguishes between geometry as an abstract system of social conventions, norms and rules, and geometrical knowledge and concepts that are individually and organically conceived. Therefore, the observer is considered as a free sovereign individual and privatised subject disconnected from a social exterior world. In short, traditional semiological and logico-analytical approaches ignore "the essentially social nature" of meaning.

In contrast to these two approaches, meaning, in this thesis, is neither considered as ordered by a closed system of self-regulated norms nor simply viewed as produced within a subject, in the interaction between one's mind and the world of objects. Instead, it will be argued that meaning - in this case

geometrical meaning - is socially produced between subjects; it has a history in the forms of production and this history is productive of thought.

Thus, in this study, moving away from the structuralist spheres of cognitive psychology and traditional semiology, I shall look towards social semiotics which incorporates cultural and social dimensions and thus, hopefully, provides new insights into understanding of geometrical awareness.

1.4 The empirical study

The empirical part of this research will be conducted through a study with three very different groups of people, who vary strikingly in their geometrical experiences as well as in their socio-economic and educational backgrounds. They will be interviewed in depth in order to examine their interpretations of a range of geometrical objects.

Two types of strategies will be used to obtain information. In the first, participants will be encouraged to express their own views about geometry by means of a series of open-ended questions. In the second, they will be asked direct questions about their views of mathematics, and, in particular, of geometry. In doing this, I want to explore how participants construct meanings. My expectation is that the two situations might produce conflicting sets of meanings.

In order to examine the various descriptions and interpretations of geometrical elements, four settings have been chosen to motivate the empirical work. The main purpose of the settings is to provide different contexts in which various three-dimensional objects and pictorial figures of different forms and sizes can be discussed. Special attention will be given to five geometrical objects: a cuboid, a cube, a cylinder, a sphere, and a hemisphere. The choice of these geometrical shapes was inspired by the constituent elements of the architectural design of the central area of Brasilia, called the *Plaza of the Esplanade*.

By taking Brasilia as a basis for the settings, I aimed not to concentrate on the axiomatic aspects of the canonical 3-D forms but rather to examine three-

dimensional geometrical objects as cultural artefacts which are differently positioned in people's everyday lives. Indeed, although I am talking about geometrical concepts and relationships I do not intend only to see them in a logical and formal context. Rather, I shall draw up a map of meanings, as signs that are read in different ways, as products of background, experience and social motivation.

At this point, my research questions may be described as follows: Given that subjects are designated as sign-producers, how do they cope with the diverse settings for geometrical objects? What do they do? What meanings do they make of the situations? My hypothesis is that the way in which the participants engage with the settings will depend upon the resources they have available where by availability I mean a relation to their sociocultural locations.

1.5 Summary

I have chosen a socio-semiotical approach as a major 'route of reading' to understand how different people conceptualise visual texts. This study, then, is an attempt to explore some key possibilities that a socio-semiotical approach can open up in the understanding of geometry.

The central concepts allowing this shift are borrowed from Eco's notions of sign-functions and his criticisms of iconicity, Kress's notions about sign and Bakhtin/Voloshinov's notion of heteroglossia together with his understanding of sign and reality. My motivation in incorporating these concepts within my research is that they offer a more complex view of meaning. Rather than regarding meaning as something contained within the text or in the reader's schemes, they draw attention to the possibilities of ambiguity and contradiction. Meaning is conditioned mainly by the social organisation which produces and reproduces the diverse and multiple ways of constructing modes of thinking, feeling, and acting towards the world. Therefore, we can trace links between a

diverse and contradictory social domain and a multiple and contradictory subject (Britzman:1991).

Moreover, in contextualising meaning, it is possible to say, for instance, that one signifier - in our case, geometrical material or pictorial figures - can, given the context, denote various signifieds. What clearly underlines this assumption is that meaning changes with the situation and context. Based on the supposition that signification is not produced in a uniform way but according to the 'material' of the signifier, I shall examine, for instance, how different visual media may motivate certain interpretations and inhibit others. In asserting that the medium may provoke or constrain some interpretations, I am not saying that meaning is an effect of the medium; rather, I would argue that it is fundamental to examine the social and cultural contexts which ultimately guide the way people 'read' the elements in the medium.

This framework will influence the analysis of the empirical part of the research. The participants' use of language for describing and interpreting the 'visual texts' (images and objects) presented to them will not be interpreted merely as words used to identify a semantic referential concept, but rather, their language will be seen to transmit values, practices and experiences associated with those images and objects.

Motivated by these concepts and ideas drawn from Bakhtin, Eco, and Kress, I can distinguish the following aims for the empirical part of my study:

- (i) to map the interpretations of the various geometrical objects
- (ii) to examine how and why they vary
- (iii) to relate these variations to cultural/social factors and to the relationship between formal and practical knowledge
- (iv) to examine how different visual media may motivate some interpretations and inhibit others

In investigating these four questions, I hope to provide an alternative framework for research in geometry. This new perspective will enable us to enquire into the dynamics of visual experience as a process of communicating and of signifying.

And this relationship will itself be dependent on the material conditions and the contextual dynamics in which the meanings are constructed.

2 The development of a socio-semiotical perspective

This chapter is divided into four broad sections. In the first section I discuss the principal fields of interest within the logico-mathematical approach to the study of geometrical understanding. After a summary, I include a discussion of the limitations of this approach. Next, in the second section, I offer a review of studies in the field of mathematics education which have argued that mathematical knowledge is socially constructed.

The third section describes the main antecedents of my theoretical position which arise from a semiological viewpoint. I consider in this section what visual semiotics has to say about the linguistic model and the implications of this line of thinking for the socio-semiotical approach to be adopted. Finally, I present my own theoretical framework and the conceptual-analytic tools to be used in this research, drawing on the ideas of Umberto Eco, Gunther Kress, and Mikhail Bakhtin.

In the last section, I discuss the relation between this theoretical underpinning and the empirical study.

2.1 Review of logico-analytical research into geometrical understanding

The connection between the physical and mental worlds is a recurrent issue discussed in the field of geometry education. A Piagetian perspective, frequently a central pivot of this debate, seeks to explain how reflection and action towards the physical world dialectically interact to construct geometrical awareness. In examining this relation, the "play" of mind upon objects has been focused upon as the stimulus for creating reflections in an individual mind. Highlighting the internal process of geometrical construction, many mathematics education researchers have examined what Stephen Toulmin has called "the inwardness of

mental life": 'the construction of experience as occurring deeply within and bounded by a self' (quoted in Bordo 1987:49). By doing so, they have emphasised the idealist notion of isolated perception and thus chose to produce an image of the 'observer' as an autonomous and unitary individual and as the source of meaning.

Despite this view of mathematical activity as primarily a constructive process in which the learner is a protagonist in establishing meaning, cognitive researchers add a further qualification. For them, meaning, as Buckingham (1993:13) has pointed out, "is still largely seen as something contained within the text, which can be 'objectively' identified and quantified. Thus, the text itself is typically defined as a 'stimulus' (...)". The effect of these inferences has been to largely neglect the possibility that different social and cultural contexts and experiences may shape ways of constructing geometry. There has been little emphasis on identifying or explaining how variation in material conditions and social relations might affect both people's understanding of geometry and their way of thinking geometrically.

In this section, I try to examine how the above tendencies are present in the issues discussed by logico-analytical researchers. The most debated theme within the field of geometrical education concerns the perceptual and representational aspects of geometrical thought. The process of visualisation is a second issue, where the dominant theory is that visual information is controlled by the mind. The third theme examines the relation between external and internal representation. The fourth reflects on what function of visual information is instrumental in producing geometrical awareness. The fifth and the sixth issues refer respectively to how a learner might articulate their experience when interacting with geometrical objects and the conflict and tensions between the image given by a formal definition and the image understood by the student. Relatively different from the previous issues, the seventh issue focuses on the role of social and cultural factors in interpreting three-dimensional shapes.

2.1.1 A dualistic scheme: the perceptual and the representational levels

Reviewing the discussion of geometrical awareness by logico-analytical researchers, it is explicitly or implicitly clear that the central concern is the polarity of perception and representation. These two terms are however not treated as unrelated elements. Indeed, from a developmental perspective, they are frequently viewed as two poles of a continuum between which attempts are made to establish a sequence of levels of geometrical thought. The presence and influence of Piaget's work within this debate is striking.

In considering the apprehension of space and geometry as a developmental process, Piaget has asserted that this understanding has various stages, beginning on a perceptual level and continuing to a representational one. At the perceptual level, the conceptualising of the objects is based upon what one sees, the external surface of the objects. The representational level involves the identification of geometrical objects in their absence, that is, when the character of shapes can be evoked without "real" stimuli. But it also operates in their presence, and is parallel to perception. Thus abstraction and logical operations are implied: one may recognise a given figure as a triangle and extend this to the entire class of comparable shapes not currently perceived (Piaget and Inhelder 1971:17). In explaining how the recognition of shapes occurs on these two levels, Piaget and Inhelder (1971:28) made a further assertion:

In the case of visual perception, shapes are recognised through an almost instantaneous structurization (...), as against this, the visual image of such shapes presupposes a mental representation, conjured up while the object is out of sight.

The transition from one level to the other occurs as a continuous process of organisation and reorganisation of structure; each organisation integrates with the previous one. Mind - as a basis of reflection and abstraction - plays a fundamental role since it acts upon the physical environment and adapts to it by a

series of 'assimilations' and 'accommodations'. This is a process of generation in which structures change systematically as the child develops: passing from a simpler structure to a more complex structure (Piaget and Inhelder:1971). According to Piaget, this structural metamorphoses means development, shifting from early concrete, undifferentiated context-bound thinking to later abstract, differentiated, generalised thinking.

This capacity for constructing mental images as a product of the interiorization of intellectual acts that are subject to a developmental process has been the pivot of theoretical reflection and empirical research with regard to geometrical education. This is especially true when the central focus of the work has been the role of visualisation.

2.1.2 Visual information controlled by the mind

Various mathematics education researchers have endeavoured to identify how students visualise and communicate information about objects in two- and three-dimensional space. Discussions about children's and adolescents' visualisation, how we examine and identify visualisation, and ultimately what visualisation actually is, have formed a central core of work.

Although there is no consensus among mathematics education researchers about the meaning of the term 'visualisation', sometimes called 'spatial ability' or 'visual behaviour', it is generally conceived as the "ability to represent, transform, generate, communicate, document, and reflect on visual information" (Hershkowitz 1990:75) or as defined by Lean and Clements (1981:267-268) "the ability to formulate mental images and to manipulate these images in the mind". The major premise of the debate on visualisation is that it is "important not only for its own sake but also because the type of mental processes involved are necessary for, and can transfer to, other areas in mathematics" (Hershkowitz 1990:76). Therefore, the concept of visualisation - as an internal

representation - is adopted in order to explore the various features of students' spatial and geometrical mental processes.

The research of Bishop, Dreyfus, Ben-Chain, for example, presents such a vista - of visualisation that is situated in the mind of individuals. All of them argue that there are different ways in which intellectual operations interpret visual information. By investigating these, Bishop (1983) identifies two types of spatial abilities: the ability to interpret figural information (IFI), and the ability to undertake visual processing (VP). The first relates to the "ability to mentally manipulate, rotate, twist or invert a pictorially presented stimulus object"; the second concerns "the comprehension of the arrangement of elements within a visual stimulus pattern and the aptitude to remain unconfused by the changing orientations in which a spatial configuration may be presented" (Bishop 1983:182). This analysis explicitly decrees that the measure of ability is the mastery of order and control of visual information. Possible differences and conflicts with associations and articulations of visual information due to lived experiences are neglected.

Concerning an individual's ability to perform intellectual activity, Ben-Chain, like Bishop, decontextualises spatial ability from everyday practices. It is the mind and eye and not the social position of a human observer that is at issue. He distinguishes three categories of spatial ability: spatial perception; mental rotation; and spatial visualisation. These categories refer to skills in representing, transforming, generating and recalling symbolic, non-linguistic information. Here, again, we see that the rational order and control of the material - the visual stimulus - is a powerful indication of competency and skill. A similar perspective is offered by Dreyfus and Eisenberg in their essay *Spatial Reasoning: Stages of Development* (Dreyfus and Eisenberg:1983). Their arguments suggest that an acquirement of knowledge about spatial reasoning ability must contribute to how one develops one's ability to visualise in three-dimensional space. In this way, through teaching, students may "visualise

mentally three-dimensional objects which are present to them by means of two-dimensional graphical representations" (ibid:241). Assuming that stages of development take place over time, all three authors present instructional sequences in which levels of development should be identified. These levels draw upon the model of Van Hiele.

Van Hiele levels have served as recurrent frame of reference for a great number of research contributions in geometry education in the 1970s to the 1990s. They were used to support educators' concerns with the visualisation and conceptualisation of geometric objects. Based on Piaget's postulates of *representational space, mental image of 'real space', perceptual processes and mental transformations*, Van Hiele, in 1957, presented a model for the development of geometric thought. Beginning with his experience as a teacher of mathematics in secondary schools in the Netherlands, he postulated the existence of sequential levels of geometrical thought to be applied in different phases of instruction in order to help students progress through geometry. This model became an important tool with which to demonstrate geometrical structure (Hershowitz:1990; Hoffer:1983).

In order to concentrate on the analysis of successive forms of geometrical structuring, the polarity between external and internal representation has gained a significant emphasis. Based on this duality, researchers have examined the process of constructing understanding of geometrical concepts.

2.1.3 The relation between external and internal images

According to mathematics education researchers, internal representation corresponds to mental images of visual information, images that are conceived individually. External representation, on the other hand, corresponds to a geometrical 'drawing', which is understood as a formal definition, conceived as a collective convention socially adopted and accepted.

In relation to external representation, 'images' (which correspond to the above mentioned 'drawing') would usually display two virtual functions: one refers to the possibility of transmitting explicitly, in an immediate fashion, the properties of the object - that is, the discourse about geometrical objects can be primarily constructed upon the 'visible' on structural elements of figures. The other function refers to the provision of support for analytical reflection through which the visible starts to become irrelevant. At this point, abstraction becomes the most important aspect in the process of mathematizing figures. The mathematical study of the process of constructing inner mental images has the external aspect of representation as a point of departure.

Internal representation develops through action and reflection by the subject ('knowledge constructor') on the object (the drawing). That is, as one moves from the perception to analytic, she/he becomes able to construct 'mental images', a process which reflects conceptual thought: the former, from direct contact with the object (the 'drawing'); the latter, from a distancing from the physical object to an abstraction as a result of logical reasoning.

According to mathematics education researchers both functions - external representation and analytical effort - are equally important in cognitive development. Emphasis, however, is given to the second function.

2.1.4 The function of the image in the process of producing geometrical awareness

By privileging abstract thinking over a perceptual knowledge that is based on actual reality, most mathematics education researchers have demonstrated a tendency to focus exclusively on the inward composition of external/internal representation. In concentrating so absolutely on the internal aspects of geometrical knowledge and concepts, they tend to consider that meaning resides either in the visual model or in the schemes of the viewer. The question as to

whether meaning emerges from what is seen or whether meaning is produced by the mental process of the viewer, is central (see for example Love, Kaput and Dreyfus).

When Love (1994) debated the functions of visualisations in learning geometry, he argued that questions about the relationship between mental objects and physical images have hitherto not been properly discussed (Love 1994:125). In relation to physical geometric images, the question is raised as to whether they represent something, and if so, what? As for the status of mental imagery, a term defined by Dreyfus (1994:3) as 'mental images of visual information' (and corresponding to Vinner's concept image), Love stressed that philosophers, psychologists and cognitive researchers disagree over whether things such as 'pictures in the mind' can exist independently of thought or language or whether they exist at all (Love 1994:125).

Attempting to clarify the status of the figure and whether it could be identified with images, Love analysed various approaches to geometrical drawings. He considered the work of Parzysz (1988), who made a distinction between a geometrical 'drawing' which is a signifier like other kinds of signifiers, such as a written text and a 'figure' (which is the geometrical object described by the text defining it). Love also analysed the work of Strässer (1991), where the 'drawing' can be used to consider logical relationships like spatial analysis and descriptions of construction.

In order to argue that these accounts do not clearly reveal the mental status of the 'figure', Love invoked the Piagetian view that concept and image are linked: "before it is formalised, geometrical thought is accompanied by some form of mental imagery and (...) these images have a symbolic function, representing, in some sense, concepts". He added that geometry is the only field in which the imagined form and content are homogeneous (Love 1994:126). Interpreting Piaget's statements, Love made this connection: a physical geometrical image

may, simultaneously, be an actual object *and* its real representation, image - that is, both signified and signifier.

Instead of accepting this point of view, Love adopted a suggestion originally made by Pierce that the mental image is the third term of a ternary relationship and mediates between signifier and signified. This is the form proposed by Gattegno (1989, quoted in Love 1994:126) and extended by Tahta (1981, quoted in Love 1994:126), as the triad 'action-image-thought' which, "although distinguished for analysis are experienced simultaneously". In this,

image is a mediating link between action and thought. Our perception of objects yields some inner mental activity which we can refer to as an image. Our awareness that we can actually act on objects *and* perform virtual actions suggests that we can operate on these images (...) and that such images can become (...) stable to be independent of the objects from which they arose (Love 1994:126-127).

For Love such vividness of imagery as elaborated by Tahta "enables us to think of mental images as concrete objects". By recognising inner states when 'we might be said to be 'imagining'', Love discusses the role of physical stimuli in geometrical thinking by means of visualising through software.

Before presenting his argument about reasoning with diagrams, Dreyfus (1994) insists that the *diagrammatic* is the most important type of visual information in mathematics. When representing a mathematical structure, diagrammatic visual information implies two mappings: from the mathematical structure to the diagram, and from the diagram to the mental image. According to Dreyfus, while mathematics deals with external representations, through diagrams, the psychology of mathematics education is concerned with internal representation (or mental images) corresponding to diagrams. He then claimed that mental images (or visual imagery), rather than the diagram, directly influence reasoning processes.

In order to understand how diagrams and operations on diagrams influence the learner's construction of visual images, Dreyfus investigated the formation and properties of visual images by means of some broad questions: the analogical

(pictorial) vs. the propositional (sentential) nature of the visual image (diagrammatic information) when stored in our minds; our mind's storage of pictures or diagrams *vs./and* storage of interpretations of diagrams; and the consequences of these variants for the mental processing of visual images. Dreyfus (ibid:7) examined the long-lasting controversy which has existed among cognitive scientist between two opposing points of view: Is diagrammatic information stored in our minds analogically or propositionally? What this seems to indicate is the frequent difficulty in separating external representation from internal representation in the understanding of the diagrammatic (visual) information.

Nevertheless, Dreyfus comments in a general way on "the classic view of imagery as internalised perception": "Many spatial mental representations appear to be based on our conceptions of the visual-spatial world" (Dreyfus 1994:14). Although he suggests that actual reality may affect the way people conceive visual information, this reality is abstracted by him and indeed by the majority of researchers who take Piagetian constructivism as their main reference point.

Kaput (1994) is concerned with exploring the 'visual and imaginistic aspects of cybernetic manipulatives' in attempting to examine how they may be used to support mathematics learning. This discussion is based on an interactive perspective which postulates a distinction between two types of activities in mathematical experience, mental and physical.

Kaput stresses that in order to develop projects of 'notations environments', either such traditional physical manipulatives as multiple blocks, Unifix cubes, or cybernetic manipulatives, one must understand the complexities of the mental and physical representations. Visualisation underpins his discussion. Kaput (1994:167) points out:

I believe that understanding the coordination between concrete, physical, visual activity and mental activity is an important, if not the key, context in which to

understand the visualization process. (...) I make the standard assumption that visualization is rooted in visual perception, but that its actual function is controlled by semantically defined mental structure, sometimes referred to as mental schemata. In other words, I am taking the position that, at least relative to the doing and learning of mathematics (or perhaps more narrowly, school mathematics) a form of schema theory applies to visualization.

Mathematical knowledge is not something inherent to us but rather the result of construction is implicit in Kaput's remarks: a self-building structure operates, that is, the subject builds knowledge by her or his own activity.

2.1.5 The articulation between a subject and geometrical objects

Much of school mathematics discourse concerns visual information, that is, visual models - 'solids or drawings' - are created with the purpose of illustrating concepts and definitions. In geometry, the connection between visual experience and actual matter is so strong that there seems to be a symbiosis between the two - that is, images appear to convey information.

Thus many understand the attributes of geometrical concepts as visual entities. Likewise, some consider that the process of interpreting and reacting to a geometrical figure is a matter of discovering meanings within a visual text, its interior 'having *intrinsically* conceptual properties' (e.g. Fischbein 1987,1993; Dufour-Janvier et al 1987; Laborde 1990; Duval 1994; Mariotti 1994a 1994b). This tendency is demonstrated by a remark of Fischbein's (1987:104):

The concreteness of visual images is an essential factor for creating a feeling of self-evidence and immediacy. A visual image not only organizes the data at hand in meaningful structures but it is also an important factor guiding the analytical development of a solution.

When mathematics education researchers analyse the interaction between concept and image two main issues arise, time and again: first, the functionality of images in the process of constructing geometrical concepts; second, the conflict and tensions between the image given by the mathematical definition and the image

understood by the learner or the internal image that the learner constructs. By examining the articulations of the subject upon geometrical objects, it is noticeable that most mathematics education researchers conceptualise this relation in terms of the transmission of information. This transmission model involves a linear communication between an individual subject and an individual visual text in which the subject is an active decoder of the message. Her or his role, therefore, is to extract the semantic properties of the visual text; and the function of the text is to stimulate the inner images of the subject.

The concentration of analyses on generic 'communication' or 'tensions' underlying these described and internally constructed images again suggests a tendency to focus on the concept-image relation at a high level of abstraction.

2.1.6 Conflicts between external and internal images

Mariotti (1994a), in her paper about images and concepts in geometrical reasoning, illustrates well the Piagetian point of view as seen in Love's work (1994). She claims that the interaction between concept and image is such that geometrical reasoning can be considered as a dialectic process involving two components: the figural and the conceptual. The notion of figural concept adopted is that of Fischbein according to which the relationship between images and concepts in the geometrical domain concerns both the internal (mental level) and the external (representation). Examining this relation, Mariotti is concerned with the analysis of cognitive 'conflicts' in the construction of geometrical knowledge. 'Conflict' appears between the external image ('drawing') and the learner's signified (internal image). The frame of reference is in the 'drawing', that is, the direction given by the mathematical (geometrical) definition. Thus, the 'drawing' is more perceptible, more 'palpable' and hence more useful in measuring knowledge. It is at this point that the signifier gains importance since the external representation (or 'drawing') provides the scheme to measure

'deviations' from the levels conceived by the Piagetian/van Hiele's geometry (from the perceptual to the abstract).

The point of attention here is to consider a student's competence in conceptualising the geometrical entity as formally given. Here, the word 'competence', not only denotes the capacity to identify and decode certain conceptual properties 'intrinsically' present in the visual model, but also the capacity to integrate the 'conceptual and figural properties in a unitary mental structure'. This capacity is considered, by itself, to be the condition for a complete awareness in geometry.

In examining this issue, Vinner (1983) and Vinner and Dreyfus (1989) works with two concepts: *concept definition*, related to formal mathematical definition and *concept image*, all kinds of mental representations of the concept together with associated background knowledge reflected in the individual mind. The author describes the difference between a concept definition (or definition) and a concept image (or image) as follows:

The student (...) does not necessarily use the definition when deciding whether a given mathematical object is an example or non example of the concept. In most cases, he or she decides on the basis of a *concept image* (...) the mental pictures associated (...) with the concept name, together with all the properties characterizing them. (...) Hence, the set of mathematical objects considered (...) to be the examples of the concept is not necessarily the same as the set of mathematical objects determined by the definition (...) (Vinner and Dreyfus 1989:356).

In asserting this, the author recognises that a student's behaviour may differ from the teacher's expectations (an anticipated coincidence between the concept image and the concept definition), and he poses the need to examine the existence of common cognitive levels among students.

In order to examine cognitive processes, Vinner and Herszkowitz (1980) propose strategies for categorising and classifying the way students and teachers understand geometrical concepts. They propose certain features, suggested by research results, that illustrate types of judgement in students' and teachers' concept images of basic geometrical concepts. According to these authors, these

features are based mainly on prototypical examples that serve as models for their judgements. The types of judgement proposed by them were related to a sequential order in the attainment of the geometrical concept, that starts with the perceptual (visual) and moves to the more abstract (analytical) (Herszkowitz:1990).

Within this type of analysis, geometrical knowledge is often universalised. In so doing, researchers neglect to examine important questions such as: how different interactions and patterns of seeing a geometrical figure may be affected by everyday social practices or how 'inner images' represent ideas and social concepts.

Even some researchers taking a social approach to logico-analytic orientations have been unable to address these questions. This, as mentioned earlier, is because when failure to understand a mathematical task is explained, researchers seem to return to theories of cognition based on cultural deficit. Yet, there is evidence that factors of culture, experience and familiarity with conventions formulated by Western culture to express/represent space have a wide range of effects on the ways individuals, or groups, communicate and interpret information on three dimensional shapes (Herszkowitz 1990:78). I will summarise these studies in the next section.

2.1.7 Cultural Effects

A study by Mukhopadhyay (1987) provides a good example of the cultural effect of work experience. She carried out empirical research in an isolated Indian village with 8-12 year-old children who had almost no schooling and had not been exposed to the conventions, common to Western culture, of representing the solid objects presented to them. The purpose of her study was to examine whether the children's visual representational ability was related to their apprenticeship training in the family occupation (their culture). The results

pointed to the conclusion that potters' children who had worked with 3-D solids produced more complex representations than weavers' or farmers' children. Mukhopadhyay seemed however, to ignore the cultural consequences of this finding, returning, in her conclusion, to a more psychological explanation.

Mitchelmore (1976) reviewed cross-cultural research on concepts of space and geometry and compared perceptual abilities among different cultures: Africans and Europeans. He argued that Africans of all nationalities appeared to be considerably "retarded" [sic] in perceptual development when compared to Europeans and North-Americans of the same age and level of schooling. In order to relate perceptual development *versus* geometry achievement, he concentrated on research developed through non-verbal intelligence tests; block design and embedded figures tests; a Piagetian test; paper-and-pencil tests of spatial ability; and concluded that cross-cultural differences were due to factors like physical environment; social environment; cultural influences; nutrition; and skin colour [sic].

Similar results - a deficit in geometrical awareness among students in developing countries - were found in two other studies by Mitchelmore (1976, 1980, 1983). By comparing students from urban areas of Jamaica and West Germany in one study (1983), it was demonstrated that the geometrical knowledge of Jamaican grade 9 students in secondary schools was inferior to that of grade 5 students in German schools. In another study (1976), it was found that the spatial ability of Jamaican grade 8 students was equivalent to grade 5 German students.

Mitchelmore's main interest was to examine the influence of culture on spatial ability and geometrical awareness in developing and advanced industrial societies. We can identify at least two kinds of problems with his approach. The first may be summed up as the problem of the term "cross-cultural research" itself. This term seems to suggest a presumption of an integral and organic culture, that is, a tendency to represent persons or groups of persons, who live

in a particular society, as a whole, and to ignore the complex relationships that exist between them. According to the anthropologist Mitchell (1969) - who used the notion of social networks in the interpretation of field data as complementary to conventional, structural, functional analyses - "structuralist generalisations about the behaviour of people in terms of the positions they occupy in the social system" are "based (...) on abstractions [that] ignore individual deviations from the pattern", and it is these deviations which are "essential elements of social actions (...), done by situational analysis" (Mitchell 1969:8-9).

Schapera (1938:29 quoted, in Van Velsen 1969:136) also criticised structural analysis aimed at social morphology for ignoring individual variations. In a text called "The craft of social anthropology" he stressed that:

Culture is not merely a system of formal practices and beliefs. It is made up essentially of individual reactions to and variations from a traditionally standardised pattern; and indeed no culture can ever be understood unless special attention is paid to this range of individual manifestations.

Van Velsen (1969:136) explained the above remark as follows:

(...) norms, general rules of conduct, are translated into practices; they are ultimately manipulated by individuals in particular situations to serve particular ends. This gives rise to variations for which the structuralist writer does not account in his abstract model.

Taking this criticism into account, we conclude that Mitchelmore assumes a monolithic view of the society or community studied, and thus is blind to a 'situational' analysis where group differences could emerge.

According to Bishop (1988), there is widespread recognition of the need to re-evaluate Western school educational experience, because of the educational failures of children from ethnic minority communities. He relates this problem to the mathematics curriculum and its relationship with the home culture of the child, since there is a belief that mathematics is 'culture-free' knowledge, viewed by him as a misconception. Research evidence from anthropological studies, he says, has suggested the existence of 'ethnomathematics': "mathematics must now be understood as a kind of cultural knowledge (...) which need not

necessarily 'look' the same from one cultural group to another (...)" (Bishop 1988:180). Yet according to Bishop, any mathematics educator who works in cultural-interface situations "(...) soon becomes acutely aware of the influence of value-conflicts on the mathematical learning experience (...)" (Bishop 1988:181). Through attempts at reconciliation between different cultural values with 'universal' concepts, the author tries to find a way to conceptualise mathematics as a cultural phenomenon. He proposes that mathematics as cultural knowledge derives from humans engaged in six universal activities (Bishop 1988:183-4).

Bishop is concerned with problems that made it difficult for children from ethnic minority communities to learn mathematics. In recognising the problem, he argues that in multi-cultural societies like the UK., the mathematics curriculum should not be reduced to one particular way of understanding mathematics. Each cultural group, he stresses, is capable of generating its own mathematics ideas, and that 'Western' mathematics may be only one mathematics among many.

In attempting to reconcile this 'culture-conflict', Bishop in fact proposes two, I believe, irreconcilable directions: first, he recognises that in order to establish mathematics activities it is necessary to recognise the cultural differences between groups; second, he proposes universal activities which should be "carried out by every group studied, and also necessary and sufficient for development of mathematical knowledge" (Bishop 1988:182).

In accepting a universalist view, he incorporates the idea of the 'universal subject'. In this he associates an innate disposition of mankind to learn certain mathematics topics. Bishop privileges the mental universe in which he says: "mathematics (...) [involves] six universal activities in a sustained, and conscious manner". This latter meaning may be referred to the psychology of intentions, to the realm of subjectivity which, in its emphasis on logic, excludes cultural and social differences. In other words, Bishop tries to make parallels

based on the specific evolutionary process of different groups. This assumption implies that significant regularities take place in that process and the identification of these allows one to categorise the mathematical laws ruling all groups (McLaren 1991).

2.1.8 Summary of issues raised by the logico-analytical model

By extracting students' inner images from any social context, most researchers assume that the students' notion of a geometrical definition 'reflects' cognitive 'apprehension', perceptual or representational. In investigating this 'apprehension', some of the mathematics education researchers are concerned with the analysis of the cognitive conflicts and tensions discussed above in terms of sequential or a priori levels.

In order to concentrate on analysis of successive forms of geometrical structuration, these researchers exclude any consideration of social and cultural differentiation. They assume instead that differences are a cognitive fact considered as neutral, value-free, and objective. In so doing, most tend to consider geometrical structure as a set of universal elements and rules common to all cultural practices, which they ascribe also to be the structure of the human mind that develops by a series of actions upon concrete objects.

From my perspective, this framework has some pedagogical problems: geometry teaching and learning is reduced to an issue of correspondence between internal and external representations, in which learners, individually or in groups, are classified according to cognitive levels. This cognitive 'conflict' should not be ignored. However, in emphasising the individual's internal mental process the researchers have little to say about issues related to social functions and interactions. For example: Why learn geometry and who should learn it? What happens with geometry outside school?; How do peers discuss their geometrical understanding? In other words, how might the use of geometry as a

form of language be understood as instrumental in the acquisition of geometrical knowledge, and as an activity closely related to concrete, practical experience? Thus, in the very specific contexts of practice, it is not possible to talk about geometrical cognitive elaboration without mentioning its social function.

2.2 Incorporating a sociocultural dimension

Although Piaget's constructivism is still the preeminent paradigm in mathematics education, (including geometry), in some areas of current research this model has been challenged. Criticisms have mainly involved the recognition of issues such as contextualization and the influences of social aspects of learning and teaching mathematics. Besides the recognition of the specificity of mathematics practice within different social situations, current analyses reveal that the influence of schooling does not take a uniform route and, that, under certain circumstances, the contribution of so called informal education is significant (Nunes et. al: 1993).

These dissensions question the Piagetian model's main focus, centred as it is on the individual. As the anthropologist Rosaldo (1984:138) has stressed:

Ultimately, the trend suggests, we must appreciate the ways in which (...) understandings grow, not from an 'Inner' essence relatively independent of the social world, but from experience in a world of meanings, images, and social bonds, in which all persons are inevitably involved.

Among researchers concerned with this social dimension of mathematics education, the work of Nunes et. al (1993), Lave (1988), Luria (1976), and Walkerdine (1988) is prominent. All incorporate the sociocultural dimension of mathematics education as a focus of analysis but adopt different paths. What they have in common is a belief that mathematics is not, and should not be examined in the abstract, but rather in relation to a 'real' situation. As Wittgenstein (1978) argued in his *Remarks on the Foundations of Mathematics*, mathematics is a matter of using a particular kind of language, not concerned with ready-made

objects or truths, but 'being able to go on' according to rules dictated by particular modes of life. I will illustrate this argument through the work of the four researchers mentioned above.

Nunes et. al (1993) and Lave (1988) have suggested that the investigation of mathematics also examines the society into which mathematical elements are inserted. Mathematics concerns the social and cultural context in which it is used, the social practices, the values, and the rationale of a particular community. From this perspective, there is no separation between 'mathematics' and 'the world', since reality is constituted by the modes in which a language is learned and used.

Nunes et. al (1993) compare 'institutionalised' and 'natural situation' contexts when exploring mathematical abilities. After presenting a general survey of studies involving mathematical activities, the authors centred attention on how the literature contributed to the establishment of boundaries between formal and informal mathematics. According to them, mathematical reasoning may take different forms, and any form observed in everyday activity will be an example of informal mathematics and if the methods used in problem solving are not school methods, they are termed informal methods (Nunes et. al. 1993:4).

Starting with the working definition that any mathematics practised outside school is informal, the authors set out to fill a gap in the existing literature in their contribution towards a systematic comparison of informal and formal mathematics. In attempting to establish connections, similarities and differences, among three types of mathematics - "the one constructed by children outside school, the one embedded in everyday cultural practices, and the one that school aims to teach in the classroom" (Nunes et. al 1993:5) - the authors carried out studies, in and out of school. These involved arithmetic and simple mathematical concepts covering "a wide age range and several types of people - children and adults, students and workers, urban and village people" (Nunes et. al 1993:5).

In observing the different types, they argued that mathematics is both a scientifically organised science (formal mathematics) and a human activity. Here, the researchers assume that, if we take for granted that human activity is organised, and if we accept the Piagetian notion that logico-mathematical structures may be conceived as the main reference for this organisation, we are left with many unanswered questions. For instance, to what extent does the social situation influence human activities? and what kind of differences do social circumstances make?

In addressing these questions, the authors challenge the psychological theories which have been the basis of pedagogies in mathematics, which have usually looked only at the logical constraints for solving mathematics problems. In these, the types of models that people build to understand situations - natural or imagined - and the different forms of representation that emerge from the effort to build these models, are usually neglected. There is a challenge for educators and researchers, in both psychology and education, to look at those aspects of cognition that have been analysed in order to use them in new theories of learning (Nunes et. al 1993:154).

Lave (1988) addressed attention to the connection between cognitive theory, educational forms, and everyday practice, seen as historical and cultural complex interactions in the 'lived-in world'. Through ethnographic research among apprentices, instead of trying orthodox explanations of cognitive processes, the author presented empirical investigations of the learning and use of mathematics both in schooling and in craft apprenticeship. In the context of everyday arithmetic, the work challenged the importance of learning-transfer across different situations as a source of knowledge and skill. It also raised doubts about experimental methods of investigating cognition and pointed to the need for an alternative analytic framework with which to approach the study of everyday practice (Lave 1988: xiii-xv; 1-20).

Methodological questions on how to study cognition as a social practice are also faced in the work of Luria and Walkerdine. Luria (1976)'s study *Cognitive Development - its Cultural and Socio-Foundations* recognised the important contribution produced by Gestalt psychology in relation to the knowledge of mind. However, he criticised this research on the perception of geometrical shapes on the grounds that it was mainly concentrated on delineating the basic laws of structural processes that unite psychology and physiology as the natural basis of human cognitive processes (Luria 1976:31). The problem was that social practices could not be seen as possessing a history (Luria 1976:20).

After denying the acceptance of perceptual organisation as an internal activity within the organism of the individual, Luria proposed to examine whether these laws of perception were equivalent for people of different cultures. His empirical focus was concerned with the interpretation of data collected in the former Soviet Union, from different groups who varied in educational background and experience: illiterate women (Ichkari) living in distant villages; collective-farm activists who had attended school temporarily; barely literate women, studying in short pre-school courses; and women teaching kindergarteners. He was interested in investigating the criteria those subjects developed to name and classify different geometrical figures. Luria's argument was that the nature of the practical experience of the subjects affects the mode of perceiving geometrical figures.

The data revealed that the women studying at the women teaching kindergarteners were the only group who defined geometrical figures by categorical names (circles; squares; triangles). According to Luria, this particular way for classifying figures gave the impression of being 'natural and self-evident'. In fact, Luria credited schools with fostering the ability to abstract, generalise and think scientifically. Within this perspective it was understandable that only 'the most culturally developed group' gave categorical attributes to the geometrical figures. Subjects in other groups employed

attributes essentially based on 'concrete and object-oriented names', in which everyday references were emphasised. This means that, with more formal education, the percentage of using abstract geometrical concepts increases.

Therefore, Luria stressed that the source of classifying geometrical figures depended on the cultural level, in which schooling was a determinant factor for the development of abstract reasoning. At this point, my approach cross references with that of Luria's. However, while my starting point coincides with his since he recognises that social experience produces effects upon the mode of interpreting geometrical figures and objects, Luria's work serves for me as a useful point of departure rather than as a comparative study. Like him, I agree that social experience produces effects upon the mode of interpreting geometrical figures and objects but the theoretical reflections, the methodology, and the procedures adopted for further analysis will take a different route.

The divergence with Luria's study started when I observed that, although he asserted that geometrical knowledge was the product of a specific historical and cultural context, he did not enquire about the specific processes through which knowledge was produced. Although he recognised social differentiation in geometrical awareness between women or peasants in remote villages, and women teaching kindergarten and active collective farm-workers (Luria 1976:15), in fact Luria, as a psychologist, was interested in examining changes in the structure of mental processes as a cognitive activity. In this, he stressed that:

The structure of activity does not remain static during different stages of historical development and the most important forms of cognitive process - perception, generalisation, deduction, reasoning, imagination, and analysis of one's own inner life - vary as the conditions of social life change and the rudiments of knowledge are mastered (Luria 1976:161).

In contrast to Luria, my focal point of interest is not to examine the modifications in the structure of mental and cognitive processes, but to concentrate on the social practices which produce specific knowledge. In my study I search to examine different practices - school and work - in which geometrical

elements are inserted and operated upon in distinct ways. I believe that, by doing so, I am able to analyse the more immediate conditions capable of indicating a possible explanation for any differences found while communicating and interpreting geometrical elements.

Valerie Walkerdine (1988)'s study *The Master of Reason: Cognitive Development and the Production of Rationality* - represents an important social reflection on mathematics education. She analyses the production of language as a 'discursive practice' within the daily life of young children. Her point of departure in attempting to investigate the social production of language and thinking is the argument that meaning is socially and dynamically produced.

In spite of the fact that Walkerdine does not deal with geometrical objects but with numbers, she presents a theoretical frame of reference relevant for my work, particularly in the kind of argumentation she has developed to consider mathematics education from a socio-semiotical perspective.

With an emphasis on the production of terms in the pedagogical process within mathematics learning, Walkerdine proposed that discursive practices created particular intersections of the material and the discursive signifier and the signified (the signifier's conceptual scheme on representing schemata). The same signifier may exist across practices, but this does not mean that the same signs are created (Walkerdine 1988:30). By pointing to the multiple signification of many signs within particular practices, she demonstrated how participants were positioned and regulated. The approach presented was post-structuralist. It offers a way beyond universalising notions. Besides denying the universalism of any discourse and of the sign, Walkerdine conceived of the sign as a triadic relation between world/object/action, in a complex relation of signification within a practice (Walkerdine 1988:1-9).

By working with relational terms - a fundamental part of children's acquisition of a mathematical lexicon - such as *same*, *different*, *more*, *less*, Walkerdine's data revealed that it was not possible to talk about children's

competence, or ability. For her there was only a complex set of relations of signification where the signs produced are specific to the practices themselves. Moreover, these often have a variety of significations within the relations of regulation dictated by the institutionalised practices of schooling. It is through the production of signs, practices and in their regulation, that the meaning of a concept is produced: for example *more* and *less* may not form a contrastive pair, and quantity relations may be inscribed in those practices by using terms such as *a lot* and *a little* (Walkerdine 1988:11-31).

In order to address how texts and cultural/ideological practices operate, Walkerdine made use of Lacan's psychoanalysis, Saussure's linguistics, and Foucault's semiotics. What is of concern to us in Walkerdine's approach is the way she considered the Saussurian notion of signifier/signified (including the Piagetian view of the relationship of signifier to signified as one of representation). This has implications for the production of sign systems, seen by her in terms of pedagogical practices. By using the post-structuralist work of Foucault, Walkerdine argued that language was organised concretely, that is, it is socially and institutionally motivated. Thus, she rejected any possibility of a universal, trans-historical system, in favour of specific historically-generated bodies of knowledge.

Although the four authors discussed in this section may diverge significantly on many points, all of them argue against mathematical reasoning as an independent activity that takes place between the mind and actual objects. Instead, there is a recognition of the role of the social and cultural context in structuring mathematical experience. This debate provided a starting point from which I considered the possibility of transcending the persistent faith of researchers in the inner subjective world of the mind and its role in geometry education.

2.3 Developing an alternative theoretical approach

My interest here is to consider how concepts like 'sign-functions', 'sign', 'visual text', and 'heteroglossia', might reveal ways of analysing interpretations of three-dimensional elements. By incorporating these ideas into research on 'visual texts' my intention is to move from the Piagetian perspective which conceives geometrical knowledge as a cognitive process concerned with the interiorization of intellectual acts, towards a perspective which indicates a cultural and social direction.

At first sight, the semiotical approach seemed to offer the most adequate alternative since semiological concepts, as a 'route of reading', allows us to recognise that geometrical knowledge is rooted in the material realities of people's lives. However, closer investigation revealed problems. The most visible was that anyone guided by the theoretical approach of semiotics is faced with acute problems of selection. A semiotic analysis might resort to several models ranging from linguistics to psychoanalysis, from mathematics or logic, suggesting an enormous variety of trends and orientations (Hawkes 1977:124). Furthermore, each of these schools represent and conceal different practices and divergent views. It is beyond the scope of my study to evaluate the highly complex issues in the debate between possible correspondences or conflict within the semiological fields of research. But it is important here to mention one aspect.

The problem of selection faced by a researcher taking a semiological approach arises from the lack of one single direction within the field. In fact, there is not a unique or unanimously accepted meaning to the notion of 'sign' and 'visual text'. Given this diversity of meanings and bearing in mind that the choice is determined by the overall thematic concerns of any study which are in turn linked to a particular mode of reflecting reality, I selected a model of analysis based on Bakhtin/Voloshinov (1973,1981), Umberto Eco (1979), and Gunter Kress (1993).

In order to consider geometrical awareness from a socio-semiotical perspective, I shall gradually digress from the intrinsic and self-regulating structure of the linguistic model of Saussure to an approach that reveals the subject within a social and historical circumstance. The following section is organised into three overlapping sub-sections, whose trajectory comprises ideas that go from Saussure, Barthes, Eco, Bakhtin and Kress.

2.3.1 The sign as a linguistic and as a semiotic reference

Ferdinand de Saussure had an extensive influence on modern attempts at developing a phenomenology of language and of signs. His work has been especially influential in conceiving dichotomies such as signifier versus signified, *langue* versus *parole*, discussion about which no structuralist or post-structuralist has escaped from judging, explaining or criticising. The importance of Saussure was pointed out by Culler (1976:53) who argued that it

lies not simply in his contribution to linguistics *per se* but in the fact that he made what might otherwise have seemed a recondite and specialised discipline a major intellectual presence and model for other disciplines of the human sciences.

I critically emphasise that aspect of Saussure's interpretation of language which refers to his binary concept of sign and its arbitrary nature. This exhibits, to me, a Cartesian dualism.

The central unit of the language system is the 'sign', defined by Saussure as the result of a binary unification of the signifier - spoken or written words, objects, or mathematical formulae - and the signified - the concept to which a signifier refers. In discussing the concept of sign, Saussure established the principle that the sign is arbitrary. One of the most important characteristics of its arbitrary nature is the sign's relative independence from its physical condition, *being the relationship signifier/signified conventional and unmotivated rather than natural* [Saussure's emphasis]. (Saussure:1983). Indeed, the

linguistic sign links not a thing and a name, but a concept (signified) and a sound-image (signifier).

At this point, the principle of arbitrariness becomes problematical. One of the main objections is the emphasis given to the inward structure of the sign, disjoint from the material and cultural reality in which it is produced. Saussure asserts that signs, by a differential contrast, acquire meaning within a closed system, independent of any contextualization and detached from any particular social and historical scene.

Although Saussure's theory was developed with verbal language in mind, Saussure considered that language was analogous to other modes of signification:

Language is a system of signs that express ideas, and therefore comparable to the system of writing, the deaf-and-dumb alphabet, symbolic rites, forms of politeness, military signs, and so on. [But] it is the most important of all these such systems (Saussure 1983:15).

Though Saussure's direct contribution to non-linguist semiotics is limited to these remarks, his theoretical framework of language was extended later to other non-linguistic signs, in food, fashion, mathematics, or images; all of them identified if not as *languages*, at least as systems of signification (Barthes 1967:9). Thus the universe of the semiotics became composed of a vast and multiple number of signs. A current mode of demarcation and positioning has been to frame them into two broad categories: linguistic and visual signs, where the former has been considered more complex than the latter.

Reinforcing this point of view, Benveniste (1985) has suggested that since language is the most developed and complex symbolic expression, all other sign systems derive from and depend on it. Therefore, the relation between a linguistic and non-linguistic sign system was explained by him in the following way:

No semiology of sound, color, or image can be formulated or expressed in sounds, color, or images. Every semiology of a non-linguistic system must use language as an intermediary, and thus can only exist in and through the semiology of language. [Thus] language is the interpreting system of all other systems, linguistic and non-linguistic (Benveniste 1985:239).

In a similar vein, Roland Barthes admits the inevitable inbreeding between the linguistic and the non-linguistic signs, asserting that "objects, images and patterns of behaviour can signify, and so on a large scale, but never autonomously" (Barthes 1967:10). Using Saussure's linguistic model, he formulated structural categories to study non-linguistic systems of signs and considered that non-linguistic signs like cinema, advertising, comic strips and photography only reached signification when expanded into a linguistic message. This message was the verbal language from which signifieds emerged, and from which their signifiers were named. The crucial question then was to indicate that sovereignty of the non-linguistic over the linguistic sign would be impossible. As Barthes (1967:10) stressed:

It appears increasingly more difficult to conceive a system of images and objects whose *signifieds* can exist independently of language: to perceive what a substance signifies is inevitably to fall back on the individuation of a language: there is no meaning which is not designated, and the world of signifieds is none other than that of language.

Nevertheless, whereas Saussure conceived language as "only one particular semiological system", Barthes stated that "linguistics is not a part of the general science of signs, even a privileged part, it is semiology which is a part of linguistics" (Barthes *ibid.*:11). Indeed, Barthes' proposal is that all sign systems should be seen as an extension of the linguistic. Moreover, while Saussure's linguistics, as a signifying system, flourished extensively, the situation in semiology was the reverse. Barthes considered that the main reason for the disparity between semiology and linguistics is due to the subordination of the latter to the former. Barthes thus inverted Saussure's proposal, trying to conceptualize more accurately the non-linguistic sign. This inversion was the basis of his early works - *Mythologies* and *Elements of Semiology* - which are strongly inspired by the Saussurean model of linguistics¹.

¹ However, in posterior works Barthes moves his approach deviating from saussurean's structuralism, a change that had in *S/Z*, a post-structuralist work, the most clear example of his dissociation from the traditional structuralist point of view.

Barthes's model of analysis has generated, particularly among semioticians dealing with the visual communication, arguments against his way of interpreting modes of signification through the formal principles of the linguistic model. For his theory to hold the relation of language and metalanguage, signifier and signified, and even the distinction between language and myth had to be thoroughly investigated. One of the main reasons for rejecting Barthes' model has been his failure to specify what is involved in the idea of sign as a historical and cultural product. As Ian Chambers (1974:52) pointed out:

Codes, like ideas, do not drop from the skies, they arise within the practices of production. However, Barthes reduces that production to a single moment in the process: the text; and turns that moment into a self-reflective totally divorced from its material existence.

Thus the challenge of incorporating a further dimension - the social production of signifying systems - has introduced new possibilities for semiotics research.

2.3.2 Challenging the linguistic model

Some semioticians reject the assumption that the nature of the linguistic sign comprises all non-linguistic signs. According to Chambers (1974), "Barthes equates all signs with language objects", as if all systems of signification were 'languages' (the 'language' of a film, of a dance and so on) in a clear reductionist argument. Chambers (1974:55) also stressed that:

If pictures and writing are to be treated without distinction equally as sign, constituting 'one just as much as the other', then the specificity of the practices that produced them is lost. Associated with that loss, the intentionality inscribed in those practices, as they exist within the universe of practices, is bracketed out under the blanket phrase: 'bourgeois ideology'.

According to Mitchell (1986), there have been some attempts to reject linguistic-dominated approaches, called by him 'linguistic imperialism' or by Eco (1979), 'verbocentric dogmatism', in favour of an alternative path that admits differences between signs. Kress (1993:170) also challenges the "longstanding theoretical and political buttressing of language as the primary,

most significant medium", what he calls 'logocentrism'. Similarly in Blonsky's view, the autonomy of non-verbal signs and the paths which "make the semiotic instrument stronger" were possible if we "stop using only the linguistic sign as our glasses to see the world" (Blonsky 1985:xxiii).

Another important criticism of the insistence on a correspondence between linguistic and non-linguistic signs was stated by Todorov (1973), who considered this point of view as an imposition, inhibiting the semiotics of the non-linguistic studies. For him, the two areas deal with distinct objects and linguistic concepts are inadequate to apprehend the specificity of non-linguistic ones. According to him, the reason for this inequality

lies not just in the slow tempo of science in its early stages, but also in the measure of uncertainty that surrounds fundamental concepts and principles of sign, both linguistic and non-linguistic. Semiotics is in a certain sense crushed by linguistics (Todorov 1973:19).

The notion of semiotics² then refers to some extent to the necessity of tracing a direction that seeks to understand how non-verbal signs convey meanings through a discourse not focused by the linguistic reference. This process of searching for new modalities of analysis for visual communication may be summarised in the following observation of De Lauretis (1991:207):

By abandoning altogether the hypothesis that a text or a message could be studied in itself and by means of a metalanguage, semiotics research ceased to be a kind of linguistics applied to verbal and/or non-verbal messages; from the formal study of signifying systems, [the semiotics research] turned to examine the modes of sign production and the previously ignored area of meaning (the semantical field) [my emphasis].

Within this deviation from the linguistic model, two directions may be identified. The first was to challenge the purported transparency of non-verbal signs and an acceptance that "they too are bound by effective - though not apparent - codification and rules of construction" (Bettetini and Casetti 1986:319). This may be explained through the following statement:

² The confrontation with the strong dominance of the linguistics over the non-verbal signs culminated in the decision taken by the International Association of Semiotic Studies in January 1969 to adopt the terminology "Semiotics", especially when dealing with non-verbal signs (Eco:1979).

In fact, visual likeness is not as obvious as it seems. Thus visual semiotics postulates that semiosis is as arbitrary for visual languages as for any other. In one case as in the other, the relation between plane of expression (signifier) and plane of content (signified) is not motivated (Hénault 1986:172).

The second direction is a denial of the binary polarisation of arbitrary/motivated signs captured within a formal system. This, after all, refuses the dynamics of signification and leads to a reductive account of the persons involved as self-regulated and autonomous minds. The research in semiotics has thus turned its attention to modes of sign production and the territory of meaning, or semantic field, in which both are contextualized. Here Umberto Eco's work - *General Theory of Semiotics* (1979) is of particular importance and it will be considered next.

2.3.3 Against the notion of 'naturalistic illusion' in visual text

There has been a general tendency to assume that images - 'graphic, pictorial representation, material object' (Mitchell:1986) - are expressive and thus more 'direct and unambiguous' than written or spoken words in the sense that 'seeing is believing'. Hence, a photograph of a building block or even a pictorial image of a building block, for instance, appear to indicate a real building block more straightforwardly than the sound of the word "building block", where ideas and things are less directly expressed. This position is however rejected by Hall (1985). He pointed to the existence of a 'naturalistic illusion'. Visual discourse, he argued, because of the systems of visual recognition "are so widely available in any culture that they appear to involve no intervention of coding, selection or arrangement. They appear to reproduce the actual trace of reality in the images they transmit" (Hall 1985:42). Some semiologists such as Alvarado et al. (1987), Hutcheon (1989) and Lovell (1983) also deny that meaning somehow is inherent in the image as a "moment of pure release, of pure transparency, of pure knowledge (...) with a clear mental gaze" (Krauss 1988:80).

From the 'realist' point of view, an image would have a transparent and fixed relation to the object it represents. Questioning this takes us to an examination of the relation between image and reality which has been a central focus of the debate on iconicity. Eco (1979) broadly discussed the issue of iconicity, criticizing the traditional way of presenting a problem which starts from the premise of an equivalence between image and object.

In semiotics, this perspective was sustained and reinforced by the idea of referent, quoted in the literature of the first quarter of the century by Ogden, Richards and principally by Frege, who proposed a model for the study of signification through systematic semiotical analysis called metamathematics (Kevelson 1986:532). According to this model, the term referent (*bedeutung*) was the real and actual object to which a sign can refer (Eco 1979:61). The central point of criticism of the Fregean perspective is that it perpetuated the idea that the signified of a term (whether verbal or visual) had an immediate and direct connection with the term to which it referred. This is what Eco (1979) called the 'equivocal of the referent', or 'the referential fallacy'. For Eco, this *bedeutung* should be eradicated from any semiotics investigation since it was an obstacle to understanding signification within a process of cultural contextualization.

In justifying this point of view, Eco argued that *bedeutung* did not simply refer to the real and actual object of the sign, but rather Frege's concern was marked by the condition of 'truth or falsity of the assertion' assuming therefore a strong tendency towards a logical approach. An example given by Eco which clarifies the Fregean's formal and logical point of view is the following: if we take the word /dog/, the referent (*bedeutung*) will not be the /dog X standing by me/ while I am pronouncing the word. For anyone who holds to the doctrine of the referent, this, in such cases, will be all existing dogs, and also all past and future dogs. According to Eco, 'all existing dogs' is not an object that can be perceived with the senses, but a set, a class, a logical entity (Eco 1979:66). For him,



Freagean concerns oppose the viewpoint of semiotics which is interested in signs as '*social forces*'. This is a rejection, therefore, of any semiotics for which the concept referent - as real and actual object to which the sign can refer - is the point of departure in analysing the meaning of images.

In fact, according to Eco, the analysis of any sign, including the iconic sign, should not be reduced to the referent: the referent should not be the point of convergence because it does not have meaning in itself. Indeed:

The referent is nothing but a ground or a horizon; something which can be directly experienced by means of 'factual judgement'; and may also provide the material of the sign-vehicle, but is at the same time something which is alien to the interests of the semiotics, because this discipline is concerned with the process of communication and signification of things, and not with the things themselves [my emphasis] (Bettetini and Casetti 1986:319).

In attempting to remove from semiotics the presence or even any allusion to the referent, represented in its actual or abstract form, Eco challenges two influential semiologists, Peirce and Morris, who gave special attention to the relation sign/object.

When they analysed the so-called iconic signs, both Peirce and Morris suggested that the relation between an image and its object is based on criteria of similarity such as analogy, motivation and shared properties. In this, the polarity represented by arbitrary (Saussure) and motivated (Peirce and Morris) has been considered among semioticians as the basic elements to fix the limits between image-verbal and image-visual language. This division is defined by particular modes of production and communication in which the linguistic (verbal) is manifested through codes or symbols ruled by a social community, whereas the iconical (visual) is 'direct and unambiguous' (motivated). Here, the preferred assumption is that visual signs express universalistic meaning. As a consequence, visual meaning becomes detached from its social and cultural context. Instead, the "image belongs to the order of perception", since the comprehension of images is neither based on codification or on rules of

combination applied to them; rather it is based upon the similarity between the signified and signifier (Bryson 1991:96).

In rejecting this rigid boundary separating linguistic from visual signs, Eco suggested that "similarity is also a matter of cultural convention (...) similarity does not concern the relationship between the image and its object, but that between the image and a previously culturalized content" (Eco 1979:204). In order to produce explanations of this correlation, Eco openly resists notions like 'resemblance', 'shared properties' and 'similarity to objects', concepts used by Peirce and Morris. Peirce elaborated a complex typology of signs, establishing nine major classes of signs. Among the elements presented by Peirce, Eco paid particular attention to those classified as "the sign in relation to its object", especially the icon since, according to Peirce it is not arbitrary like the symbol.

According to Peirce, the relation between the signifier and the signified in the iconic sign is not arbitrary but one of resemblance or likeness, meaning that the icon 'is a sign which exhibits the same quality or configuration of qualities as the objects denoted' or a sign is a icon when it 'may represent its object mainly by its similarity' (Hawkes 1977:128-129). Peirce's definition includes portraits, paintings, photographs, ideograms, diagrams, logical graphs and algebraic formulae, linked, for him, to functions of representation.

The meaning given by Peirce to similarity is not based on any conventional sense but, as Eco (1979;1989) stressed, relies on the concept of similitude, suggesting a logical and scientific status, in which iconicity includes similarity of abstract relations or structural homologies. Hence, "to say that a sign is *similar* to its object is not the same as saying that it possesses some of its properties" (Eco 1979:195). For example, there is not a natural correspondence between a continuous line tracing the profile of a horse and the concept 'horse'. Rather, the relation between an image drawn on a given surface and the word 'horse' is motivated by a logical relation between a given expression and a given

content; that is, the meaning is motivated by the structural and abstract representation of the horse.

Morris also gave the sign a motivated character, based, as Peirce did, on analogy. For him, a 'shared property' is the basic condition of the icon. He stressed that a sign is iconic to the extent to which it has the properties of its denotata (Morris, quoted in Eco 1979:192). Eco argues though that if one trusts in common sense by habit, the inclination might be to believe in this premise. However, it becomes obscure as common sense reveals itself to be deceptive, a tautological route. Hence, what does it mean 'to have the same properties'? Looking at a nose, we see it is three dimensional whereas its graphical image has only two dimensions. A real nose has two holes whereas the one depicted on the canvas has only two black points not able to cross over the picture screen. As Morris said:

A portrait of a person is to be a considerable extent iconic, but is not completely so since the painted canvas does not have the texture of the skin, or the capacities for speech and motion, which the person portrayed has. The motion picture is more iconic but again not completely so (...). A completely iconic sign would always denote since it would itself be a denotatum (Morris, quoted in Eco 1979:193).

Hence, according to Morris, a completely iconic sign is identical to its denotatum. For Eco, this approach, when pushed to its extreme, would lead to an annihilation of the icon as sign since the iconical sign of a nose would not be the nose portrayed but the real nose itself. Faced with this charge of rigidity, Morris reviewed his previous remark, expanding the notion of icon. He stated: "an iconic sign, it will be recalled, is any sign which is similar in some respect to what it denotes. Iconicity thus is a matter of degree" (quoted in Eco 1979:192).

Another problem with Morris's approach, according to Eco (1979), arises with the expression "similar in some respects", which may satisfy a common-sense but not a semiotical analysis, which doubts that iconic signs are for sure 'similar' to the objects they stand for. To illustrate his point of view, Eco presents the example of a glass of beer portrayed in an advertisement; one

perceives the beer, does not feel it directly but through visual stimuli, colour, spatial relationship, play of light, all of which are coordinated until obtaining the perceived structure of 'a glass with freshly poured beer'. This connection - the *perceptum* - is produced from prior experiences in which one selects and arranges the sign's components based on codes. Here the relation between the sign and the code is not concerned with the nature of the iconic sign but to the perceptual mechanism that may be a factor in communication.

A first conclusion that might be drawn is that the iconic sign does not possess the 'same' physical properties as do its object, but relies on the 'same' perceptual 'structure' or system of relations. It owns the same perceptual sense but not the same perceptual physical support. Eco states that the iconic sign looks like objects in the real world because they reproduce the conditions (that is, the codes) of perception in the viewer. He thus emphasizes that this condition of perception is selected through the codes of recognition which are *entirely* a matter of convention. Therefore, an enquiry into iconic signs should not arise from the parallelism between the "image and its object but that between the image and a previously culturalized content" (Eco 1979:204).

Indeed, Eco recognises that the "elements of motivation exist but they can only work when they have been conventionally accepted and coded" (Eco 1979:190). What this means is that he rejects the motivations of the sign in the sense of likeness implying natural resemblance to their referent. Therefore, the difference between the image of a building block and the word /building block/, for example, is not a 'trivial' difference between iconic (motivated by similarity) and arbitrary (symbolic) signs. "It is rather a matter of a complex and continuously gradated array of different modes of producing signs and texts(...)" (Eco 1979:216). Thus, he concludes saying that there is no such thing as an iconic sign: there are only *visual texts* whose pertinent units "are established - if at all - by context". (Eco: *ibid.*). And it is the code, the purposefully, established correlation between expressive and semantic units,

that decide on what level of complexity it will single out its own pertinent features.

In conceiving this flexibility, Eco (1979:49) explains that:

The sign is not a fixed semiotic entity but rather the meeting ground for two independent elements (coming from two different systems of two different planes and meeting on the basis of a coding correlation. Properly speaking there are no signs, but only *sign-functions*.

The sign-function is, for Eco, the mutual and transitory correlation of two 'functives', the elements of correlation: a 'sign-vehicle' (the expression, or physical component of the sign) and a 'cultural unity' (content, meaning). He stresses that "the same function can also enter into another correlation, thus becoming a different functive and so giving rise to a new sign-function (Eco 1979:49). From this it follows that codes change and move whenever new sign-vehicles are produced. That is, "signs are the provisional results of coding rules which establish *transitory* correlations of elements, each of these being entitled to enter under given coded circumstances into another correlation and thus form a new sign" (Eco 1979:49). Thus, semantic values vary according to the coding rules defined by the context, what Eco calls the 'mobility of semantic space'.

In recognising this mobility, Eco suggests a way to breakdown Saussure's binary structure of meaning. In dissolving the dichotomy between signifier and signified, Eco (1990:33) concludes that:

Language is caught in a play of multiple significant games; (...) a text cannot incorporate an absolute univocal meaning; there is no transcendental signified; (...) the signifier is never co-present with a signified which is continually deferred and delayed; and (...) every signifier is related to another signifier so that there is nothing outside the significant chain, which goes on *ad infinitum*.

Looking at the above concerns, there are two aspects of Eco's ideas of iconic signs that are important to my present work. In the first place, Eco challenges the realist/cartesian perspective which assumes a natural and direct conduit between sign and referent, based on the impossibility of assuming unmediated representation. This indeed is, the departure point of this research in respect to the way we might approach the cultural and social construction of visualisation.

The second aspect we found in Eco was the concept of sign-function. In that I encountered an important basis for understanding how an image, an object, and a pictorial figure may be analysed within its circumstantial and relational situation in which the meaning is constructed.

2.3.4 Proposing a new direction

Although the concept of meaning given by Eco is that of a social product, culturally constructed and contextualized, his approach is not entirely adequate to analyse the questions posed in my investigation. The reason for this partial inadequacy is that, while thinking and theorising about meaning as a social production, Eco explains it in terms of a general semiotic theory without really specifying particular signifying practices which generate meaning [my emphasis]. His method of work in semiotics, as stated by Norris (1988), claims to represent the master-science and explanatory matrix of all cultural activity, that does not permit one to locate the 'ground' of the text production, represented in my study by school and work experience taken as social practices.

For this reason, it has been necessary to include an additional tool of analysis besides Eco: one more helpful in situating and specifying different social practices as well as in understanding how these practices may generate and provoke particular signs. More specifically, in this section I shall explore the relationship between sign and reality elaborated by Bakhtin (1973) in his concept of heteroglossia (1981) and the character of sign formulated by Kress (1993), in the hope that they will offer an opportunity to deepen and reinforce the questions formulated by theoreticians of visual semiotics such as Umberto Eco. I shall stress that the purpose of incorporating an additional theoretical path as an instrument of analysis is to complement rather than exclude.

Though both Eco and Bakhtin understand the concept of meaning as a social product, culturally constructed and contextualized, Bakhtin in contrast to Eco,

challenges the idea of a general theory of language. As he says: "*the forms of signs are conditioned above all by the social organisation of the participants involved and also by the immediate conditions of their interaction* [his emphasis]. When these forms change, so does the sign [my emphasis] (Bakhtin/Voloshinov 1973:21)". Yet, Bakhtin states that "meaning is the expression of a semiotic relationship between a particular piece of reality and another kind of reality that it stands for, represents, or depicts (Bakhtin/Voloshinov 1973:28).

In order to understand Bakhtin more closely, it is important to distinguish sign, reality and ideology³ as they are conceived by him, overlapping one on another. For Bakhtin, "every physical object can become a sign" and "a sign does not simply exist as a part of a reality - it reflects and refracts another reality" (...) "the domain of ideology coincides with the domain of signs (...) [in which] they equate with one another, [that is], wherever a sign is present, ideology is present, too, [and that] everything ideological possesses semiotic value" (Bakhtin/Voloshinov 1973:9-10). Ideology is seen by him in the sense of different domains of knowledge that underlie different cultures: "the domains of the artistic image, the religious symbols, the scientific formula, judicial ruling, etc. Reality is linked to this ideology in that "each field of ideological creativity has its own kind of orientation toward reality, and each refracts reality in its own way" (Bakhtin/Voloshinov 1973:10-4). In relation to the nature of signs, Bakhtin not only particularises them in relation to the kind of social communication that is being materialised (Bakhtin/Voloshinov 1973:13), but also views them as arising only "on interindividual territory" which "cannot be called natural in the direct sense of the word: signs do not arise between any two members of the species *Homo Sapiens*. It is essential that the two individuals be *organised socially*, [his emphasis] that they compose a group (a social unit)" (Bakhtin/Voloshinov 1973:12). In another passage, he says that "a sign is a

³ In Bakhtin's view, the term ideology refers to the way in which members of a given social group interpret and understand some aspects of the world.

phenomenon of the external world", in which "both the sign itself and all the effects it produces (all those actions, reactions and new signs it elicits in the surrounding social milieu) occur in outer experience" (Bakhtin/Voloshinov 1973:11).

Meaning is, therefore, determined not by the structure of reality itself but mainly by the social organization which produces and reproduces the diverse ways of constructing modes of thinking, feeling, and acting towards the world. As the sign is equated by Bakhtin to different social realities in which it is produced, what is important to emphasise here is that different social organisations produce different relations of signification, therefore different relations of meaning. Thus if meaning is mutable, is floating, dependent on practice and context, the production of that meaning cannot be fixed and unique.

This mobility of the sign is clearly suggested by Walkerdine (1988) when she discussed the social production of language and thinking in her chapter about relational terms in everyday social practices. She stressed that:

If material phenomena are only encountered within their insertion into, and signified within a practice, this articulation is not fixed and immutable, but slippery and mobile. That is, signifiers do not cover fixed 'meanings' any more than objects have one set of physical properties and function. It is the very multiplicity which allows us to speak of a 'play' of signifier and signified, and of the production of different dynamic relations within different practices. It is for this reason that I used the terms 'signify' and 'produce' rather than represent. If social practices are points of creation of specific signs then the semiotic activity is productive, not a distortion or reflection of a material reality elsewhere (Walkerdine 1988:30).

While in *Marxism and the Philosophy of Language* the sign was seen "as the point of convergence of the individual consciousness with the social" (Morris 1994:73), in the essay *Discourse in the Novel*, heteroglossia becomes one of Bakhtin's key terms for analysing the "the continuous dialogic struggle with and between discourses" (Morris 1994:73), that is, the complex "social diversity of speech types" (Bakhtin 1981:263). According to him, language is stratified through social activity. Each activity or practice represents a specific way to conceptualise the world. "Heteroglossia" refers to these multiplicities of

discourses which represent as well, the different 'socio-ideological groups'. Bakhtin (1981:289) describes the heteroglossic features of the sign in the following way:

There is interwoven with ... generic stratification of language a *professional* stratification of language, in the broad sense of the term 'professional', the language of the lawyer, the doctor, the businessman, the politician, the public education teacher and so forth, and these sometimes coincide with, and sometimes depart from, the stratification into genres. It goes without saying that these languages differ from each other not only in their vocabularies; they involve specific forms for manifesting intentions, forms for making conceptualizations and evaluation concrete.

From a Bakhtinian perspective, multiple social discourses need not be segregated or separated points of views. Rather, as White (1993) notes, they are in "dialogic interaction in which the prestige languages try to extend their control and subordinate languages try to avoid, negotiate, or subvert that control" (1993:137). It is significant is that this perspective views discourse and the utterance, and the speaker as already social. That is, the social is not a status one receives but a condition of enunciation and meaning.

Therefore, in a Bakhtinian perspective, assertions are made by individuals but we are not free to express our own views and preferences as we please. Rather, we are social subjects inserted within socially determined formation with different backgrounds and experiences. These experiences influence the way in which we, as sign-producers, understand the world. Subjectivity thus should be grasped in its social and cultural specificity as opposed to the universal experience.

In my view, Kress takes a theoretical framework that is parallel to Bakhtin in some points. The first similarity is that Kress tries "to locate the production of signs in the social and cultural histories of the producer of the sign" (Kress 1993:173). It is at this point that Kress differs from Eco's approach.

He is concerned with the development of a new social semiotics, founded on a theory of social production and the reading of signs which I can use when

analysing how different social groups (of students and manual workers) describe and interpret 3-D elements embedded in different settings.

In his work, Kress outlines a theory of sign as a motivated and conventionalised semiotic entity. He focuses on an account of the reception/reading of signs in cultural histories; deals with the question of the boundaries of signs; and with the multi-modality of the sign, that is, that all signs are complex, existing in a number of different semiotic modes. What interests me in his accounts relate to the character of the sign and of a visual text as one of the semiotic modes.(Kress 1993:169-70;187).

By relating critical points of discordance surrounding the question of the sign, Kress points out the necessity of rethinking discourse analysis: the text seen as unproblematically established, fixed, bounded; language as an autonomous, self-contained system, discrete in all respects from other semiotic and social systems of practices (...); the assumption of arbitrariness of the relation signifier/signified as a relation of form/content in common-sense speech; and the assumption of the stable nature of the sign.

According to Kress there is an increasing amount of work about sign which criticises the traditional conception of the Saussurian sign (Halliday 1978,1985; Hodge and Kress 1988,1993; Kress 1978; Lentriccia 1980) (Kress 1993:171). This criticism encompasses what he says: "(...) sign (...) is not the product of an arbitrary association of a signifier and a signified, either from the point of view of the producer, or from a consideration of characteristics of the object (...)". [But] from the producer's point of view, according to Kress, it represents his particular 'interest' in the object, which reflects his place in the world - physically, cognitively, socially, culturally, conceptually -, an interest that is not fixed but expresses a temporary configuration of socially and culturally produced internal representation. Thus, by selecting different aspects of an object as criteria for the production of sign, the 'same' sign might be

produced differently in different moments, reflecting different interests or a different awareness of culturally existent conventions over time.

Yet for him, the object/referent is not a "relatively integral object (...)" but a highly complex behavioural event and a resultant conceptual structure" (Kress:1993:173) [what Bakhtin called 'behavioural] ideology' (Bakhtin/Voloshinov 1973:91). A person, while operating in a semiotic system (the spoken language), in which the sign already exists, "selects an existing sign which in his view most nearly expresses aspects of the signified (...)" (Kress *ibid.*).

According to Kress, the view of the production of signs in a semiotic system where the existing sign becomes the signifier for a newly produced signified has its theoretical basis in the work of the linguist Hjelmslev (1943/1969) and in the writings of Barthes (1980) and Eco (1989). Again, what Kress stresses is that, in the presence of already existing signs, the production of signs is located in the social and cultural histories of the producer of the sign, which, by its turn, is motivated by the producer's 'interest' and by the characteristics of the object. It is this 'interest' that determines the features which are to be selected and to be represented. The example worked on by Kress (1993) is illustrative of 'interest' in sign production. A 3 1/2 year-old child was drawing a car. According to Kress, the resultant sign simultaneously encoded characteristics of the sign's producer, and the characteristics of the object produced. Such characteristics for that child at that particular age are represented as the most obvious features of the sign produced - in this case the wheels of the car (Kress 1993:172). That is why he postulates that the relation between signifier and signified *in all human semiotic systems* [my emphasis] is always motivated and never arbitrary. At this point, Kress again agrees with Bakhtin when he says:

The idea of the conventionality, the arbitrariness of language, is a typical one for rationalism as a whole, and no less typical is the comparison of language to the system of mathematical signs. What interests the mathematically minded

rationalists is not the relationship of the sign to the actual reality it reflects nor to the individual who is its originator, but the relationship of sign to sign within a closed system already accepted and authorized. In other words, in algebra, completely independently of the ideological meanings that give the signs their content (Bakhtin/Voloshinov 1973:58).

Looking at the work of Eco, Bakhtin, and Kress, there can be no doubt that their approaches contrast with the subjective psychological method. They clearly offer a premise which could help me ascertain the dynamics of any visual experience, the process of communicating and signifying which is connected to material conditions and to the contextual dynamics in which the meaning is constructed.

This way of asking about meaning becomes of particular importance when I come to prepare for and to analyse the empirical part of this research. I propose that when the participants of my research used a language for describing and interpreting 'visual texts' (images and objects) presented in the empirical work, they were not only using language to transmit a semantic referential concept but they were transmitting values, practices, and experiences associated to those images and objects. Developing this assertion, I will argue that the social context experienced by distinct persons produce certain kinds of cultural knowledge.

2.4 Designing an empirical study

In this section, I shall examine the interplay between the theory and the design of the study. In other words, I want to explore how the socio-semiotical approach was implicated in decisions such as: "where to observe and when, who to talk and what to ask, as well what to record and how" (Hammersley and Atkinson 1983:45).

In order to explain that involvement in my research, an examination of the some peculiarities of the analytical concepts chosen for study seems necessary. One central concern which emerges from the concepts like 'sign-functions', 'sign', 'visual text', and 'heteroglossia', is the rejection of universalistic meaning as a product of the 'human mental process'. Of equal

relevance and importance, is the rejection that the sign, as a category of meaning, is the result of a binary unification of the signifier and signified as a closed system in which emphasis is given to the inward structure of the sign.

In rejecting the universality of meaning, and the view of meaning as resulting from a union between signifier and signified, the theoretical concepts adopted in my research strongly suggest that sign, as a mapping of meanings, is read in different ways as products of contexts, background, experience, and social motivation. What, inevitably, emerges from this assumption is the mobility, variation and flexibility of meaning.

If one of the main objectives of my research was to develop a theoretical framework for the study of geometry understanding drawing on concepts which fundamentally stress diversity and mobility, it is essential to create an empirical situation where possibilities for multi-interpretations would emerge and converge.

Therefore, in attempting to explore how people would interpret and describe geometrical elements, I have chosen three sets of people who were representatives of different background and experience. More specifically, these people differed radically from each other in relation to their geometrical experience and their socio-economical and educational background. I assumed that the way different people see visual texts might mould their own experience when making use of particular selections to describe and interpret. In other words, I was interested in the relation between the participants' interpretative strategies and their position as social subjects.

The main purpose of providing different settings was to explore a range of contexts in which various sets of material objects and pictorial figures of different forms and sizes were discussed. My expectation was that the way in which three-dimensional elements were depicted in different settings - the presence of the same shape in different forms of presentation - would produce different sorts of interpretation. This expectation was bred by the theoretical

framework adopted in my study which emphasises the contextual relationship between a visual text and its meaning(s). Underlining this assumption is an idea that meanings change with situation or context.

In contextualizing meaning, it is possible to say, for instance, that the medium may, partially, motivate some interpretations and inhibit others. In asserting that the medium-context may provoke or constrain some interpretations, I am not saying that the meaning is an effect of the medium in which the objects are inserted but rather it is fundamental to examine the social and cultural experience which ultimately governs the way people 'read' the elements contained in the medium.

This assertion is part of a general assumption that I take as a priori: it is clear that neither the physical aspects nor the location of the objects in different settings are sufficient to explore the questions posed in the present research. That is, meaning is not totally determined by the visual itself or by the structural isomorphism - the presence of the same shape in different forms of representation. Rather, I shall seek to illustrate that we are not only dealing with the way in which shapes are represented, but there also exists a "language for visual representation (...) sets of codes and conventions used by (...) [persons] to make sense of what they see" (Wollen 1982:45).

Within this perspective, we may also say that the modes of communication, the strategies and techniques used during the process of probing, have important implications for the nature of the answer obtained. Furthermore, the type of question is an important reference for the quality of the answer.

The above commentary makes it clear that the specificity of the situation has a significant role for the production of meaning but it is not at all the most important aspect in the process. As Morley (1992:100) asserts:

The situational variables will produce differences within the field of interpretations. But the limits of that field are determined at a deeper level, at the level of what language/codes people have available to them - which is not fundamentally changed by differences of situation.

Bakhtin/Voloshinov remarked upon this point, saying:

The immediate social situation and its immediate social participants determine the 'occasional' form and style of an utterance. The deeper layers of its structure are determined by more sustained and more basic social connections with which the speaker is in contact (quoted in Morley 1992:100).

In consonance with this position, my main concern was not only to focus on the situational level of producing meaning. Rather, my concern was also to analyse how social and cultural experiences intervened in the participants' interpretations of the different situations presented to them.

In order to consider all these features it was important to adopt a variety of strategies for data collection. For this reason, I planned to use two types of strategy to obtain information, one involving open-ended questions where participants were encouraged to express their own criteria; the second consisting of direct questions about their views of mathematics, and in particular of geometry.

The initial sessions involved discussions that enabled the respondents to formulate and articulate their own interpretations about the material presented to them. There was room for spontaneity and improvisation and it was hoped that this would allow the participants to fashion meaning and assert standpoints which might be linked to individual geometrical imagination. In contrast, the direct questions about geometry - such as 'what is geometry?' and 'what do you know about it?' - required an abstraction that descontextualized knowledge and skills from their practical existence since the participants were asked to theorise an understanding of geometry; provide a formal definition.

My intention in utilising these two types of strategies was to examine the connections/disconnections between the two kinds of communication. And specifically, I wanted to examine how the participants constructed meaning within the two situations I presented to them. In the next chapter, I shall discuss in detail the design of the study.

3 Methodology

This chapter begins with a broad view of the qualitative approach I have chosen. I then trace the main aspects of specific methodology adopted in the empirical study:

- (i) The participants in the study
- (ii) The settings
- (iii) The research sessions and the questions explored
- (iv) The initial difficulties in organising and analysing the data
- (v) The articulation of the analysis with the theoretical space
- (vi) The development of the categories of analysis
- (vii) The categories of analysis

3.1 Research design: a qualitative approach

The term methodology, as Harvey (1990:208) reminds us:

is not about data collection methods in themselves but is about the whole process of inquiry (...) [in which] embraces conceptualisation of the problem, theoretical debate, specification of research practices, analytic frameworks, and epistemological presuppositions.

In this same vein, Bryman (1988) says that qualitative and quantitative research, do not just denote ways of gathering data but also connote divergent assumptions about the nature and purposes of research in social sciences. It may thus be argued that these two different ways of examining particular research methods and strategies argue varying approaches to social reality and the world in general.

For our purposes, it is important to examine some differences between qualitative and quantitative interviewing. In a quantitative interview, the individuals are selected at random and the sample, which is large, stratified and precise, is considered representative. By contrast, qualitative interviewing is conducted with a small group of informants, within a specific milieu (Bryman 1988:100), through an approach not disconnected from localised domains of social experience. Here, the specificity of experience is significant for qualitative inquiry. The point is to describe richly the particular. Another key difference between the two approaches lies in the data collection procedures. The

quantitative approach tends to be associated with structured interviews and surveys. Questions are prearranged and coded to reveal patterns for subsequent analysis. In contrast, the qualitative approach tends to be more flexible and revealing, allowing negotiation, discussions and expansion of the interviewee's response.

In order to explore how individuals and groups interpret geometrical materials, I decided that the approach to be adopted should be essentially qualitative in character, thus capable of grasping information in a non-rigid way. As mentioned in Chapter 2, I wanted to adopt two types of strategies. First, a set of open-ended questions were used; secondly, I asked two direct questions about their views on mathematics, and in particular on geometry. We may say that these questions represented a quantitative strategy, precluding negotiation.

By using these two types of strategies, I expected to identify different discourses that could reflect each group's experience and knowledge of the geometrical elements presented during the inquiry.

The study observed three sets of people who differ radically from each other in their geometrical experience and their socio-economical and educational backgrounds. I assumed that the way different people see visual texts might mould their own experience when making use of particular selections to describe and interpret. In other words, I was interested in the relation between the participants' interpretative strategies and their positions as social subjects.

3.2 The participants in the study

The interviews were conducted individually with three groups of people. The first source of data was taken from two mathematics teachers in two different State schools in Brasilia: one of geometry (of the secondary students) and the other of general mathematics. The second came from two groups of students, aged around seventeen, selected by these teachers. Five students were secondary students and five were at the primary level. The third source was collected from three groups

of manual workers composed of two graphic designers, two carpenters and two potters.

The group of secondary students who had been in contact with school geometry (including the geometry of solids) for over four years was chosen by a geometry teacher described as being of a high level of geometrical attainment. All students interviewed in this group were in their last year of secondary school, about to enter university. This school (called Westside) has been considered to be of a high standard, with low failure rates. A common and widespread measure of comparison in Brazilian secondary schools has been the range of students' scores in the 'vestibular', a competitive examination for entering university. The school I selected is a well-known example of a successful State school.

The second group was selected from a primary State school (called PROEM). Pupils here tend to exhibit particular characteristics such as high rates of illiteracy, and a high dropout rate. These pupils are only admitted to the school if they are backward in relation to grade/age. They were selected as representatives of a group with almost no formal study in geometry. They were adolescents aged 16-17 years old and their level of schooling ranged from first to fifth grade.

The third group was composed of graphic designers, potters and carpenters, none of whom had completed the seventh grade, and all having been out of school for at least eight years, from the time the interviews were conducted.

A striking difference among the sixteen persons composing the three groups, fundamental to the present study, was in their geometrical experience. The strategy adopted in selecting people with such differentiation is, as mentioned in the previous chapters, inextricably linked with the development of the theoretical purposes in the present research. More specifically, variations among people and contexts are fundamental for the development of theoretical categories which primarily take account of plurality and conflicts. However, the

approach proposed does not treat heterogeneity in terms of different levels of mental process, that is, 'cognitive pluralism' (Wertsch:1991). The heterogeneity that I have attempted to search out, that is, the different forms of thinking geometrically, is connected with different forms of activities and experiences rooted in social material life. In other words, I wanted to consider differences of geometric understandings between students who were in contact with school geometry for over four years, those who have had almost no formal knowledge of geometry, and those who have had contact with geometrical shapes/forms through their own work experience.

3.3 The settings

Four settings were presented, in order to examine how geometrical elements would be described and interpreted by the groups considered. The main purpose of the settings was to provide different sorts of contexts in which a set of three-dimensional elements and pictorial figures of different forms and sizes could be discussed. The material included:

- (i) a model of Brasilia
- (ii) wooden blocks with various sizes and shapes
- (iii) a photograph of the wooden blocks
- (iv) the model of Brasilia and a set of 3-D geometrical figures of different shapes modelled on a computer

Special attention was given to five geometrical objects: a cuboid, a cube, a cylinder, a sphere, and the hemisphere. The choice of these shapes was inspired by those that constitute the architectural design of the central area of Brasilia, called *Plaza of the Esplanade*. This includes the bureaucratic headquarters of the government with its major buildings: the Ministries; the Palace of Congress, the Department of Foreign Affairs (Itamaraty Palace), the flagpole, and the Federal Government Office (Planalto Palace)¹. The twelve ministries are located along the monumental axis with six on one side and six on the other. Each is a long,

¹ The Palace of Justice was deliberately excluded for a practical reason: its shape displays the same design of both the Department of Foreign Affairs and the Federal Government Office.

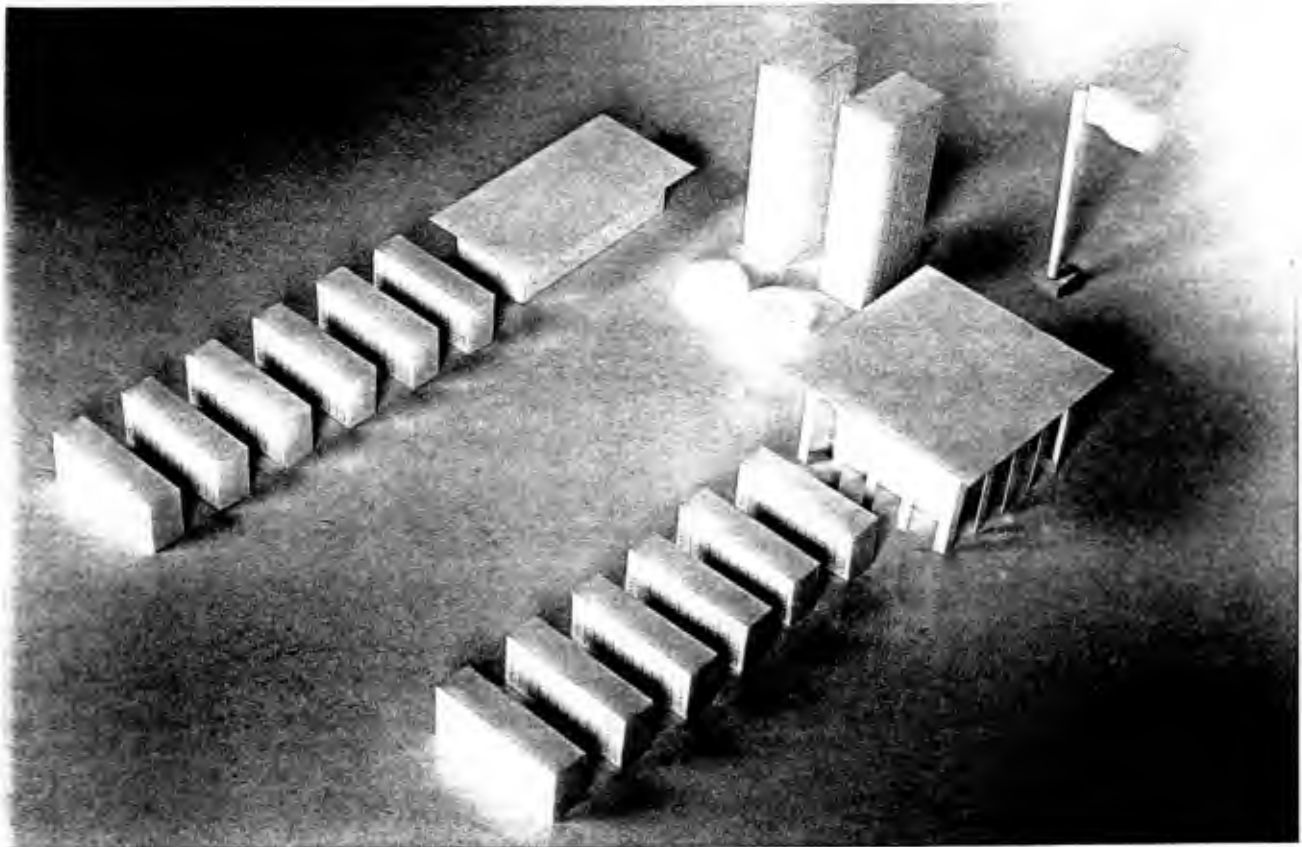
box-like structure. In the background there is The Palace of the Congress, two twin buildings, taller than any of the others. In front of the Palace of the Congress there are two cupolas, one of the Senate Chamber (convex) and one of the Deputies's Chamber (concave). In the right background is the Department of Foreign Affairs, its main body has a cuboidal structure. On the left side is the Planalto Palace which has a rectangular structure. The flagpole, placed behind the Palace of the Congress, has a cylindrical shape.

In linking this geometrical peculiarity with my research design, I had to be cautious. For instance, it is valid to state that forms such as cylinders, hemispheres, spheres, cuboids, and cubes define the form of the architecture in which the forms made by lines, surfaces and bodies seems to be boundless. Nevertheless, the fact that the architecture of a building may be described in a geometrical code (and not necessarily one of school geometry) does not mean that the architecture itself merges into geometrical code (Eco 1989:311). In this manner, my urging that the participants of my study recognised or evoked a building or a wooden block in a geometrical sense presupposed certain directions motivated by the study's design.

3.3.1 The model of Brasilia

The model of Brasilia, the first setting and the basis of all the others, contained geometrical pieces constructed from white cardboard paper, except for the hemispheres (see next page).

Figure 3.1 Model of Brasilia



The material used for the model was white polystyrene. The use of white paper was intended to avoid any colour effects that might influence the person's judgement while still describing and associating the elements observed. Each geometrical shape included (except for the sphere, which corresponded to the fitting of the two hemispheres) corresponded to one element of the architectural structure of part of the central area of Brasilia city, or the *Plaza of the Esplanade*. In the Model, a 59x27 inch paper was used as a platform in the same arrangement as in the real setting.

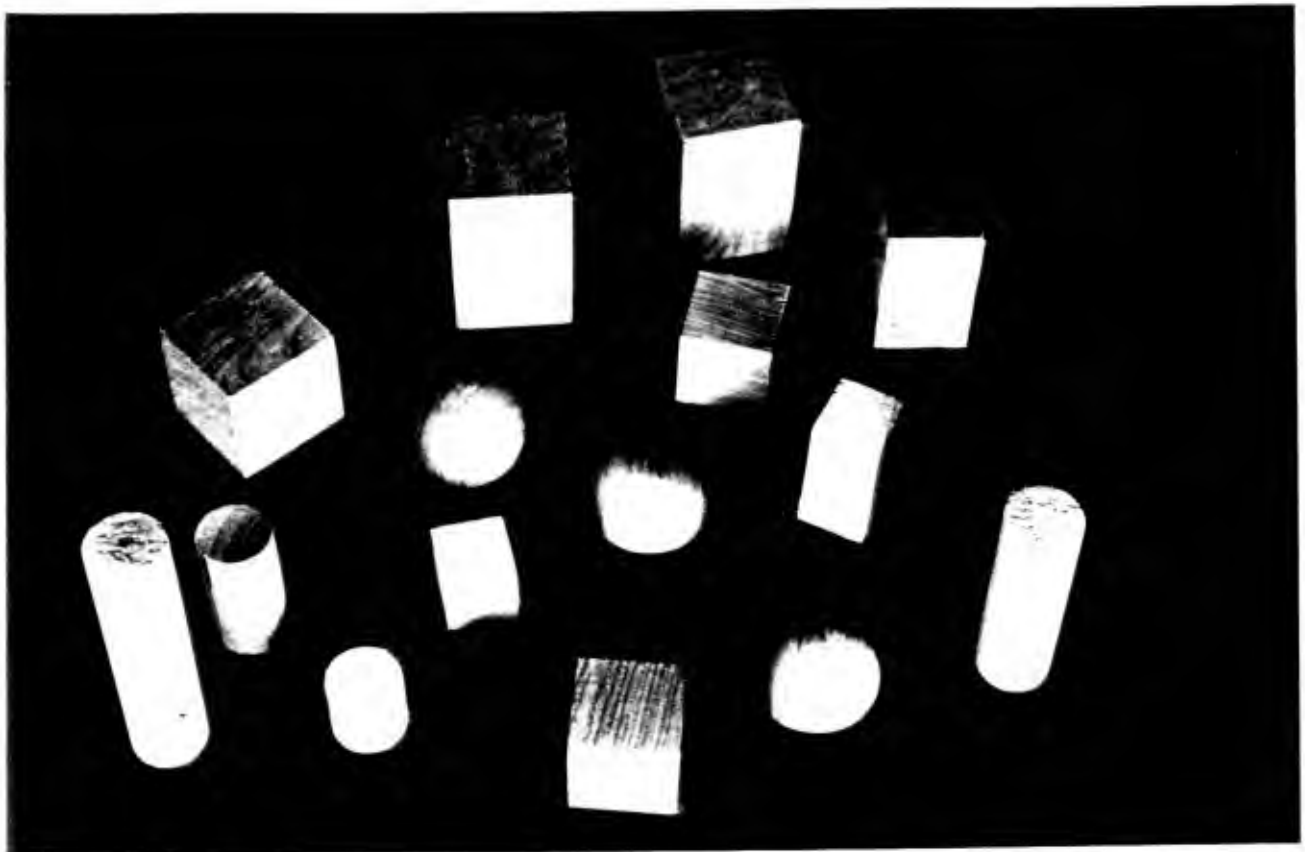
3.3.2 Wooden blocks

This setting consisted of a set of wooden blocks - of natural colouring - which varied in size, shape and volume. There were four cylinders, four cubes, four cuboids, two hemispheres and one sphere.

3.3.3 A photograph of the wooden blocks

In this setting, a photograph of several of wooden blocks was shown. The elements contained in this setting were the same group of wooden blocks (see figure below).

Figure 3.2 Photo of wooden blocks



3.3.4 The model of Brasilia, and a set of 3-D geometrical figures of different shapes modelled on a computer

In this setting a group of three-dimensional geometrical figures and a model of Brasilia was modelled on a computer. The area depicted was the same as used in the initial context, *i.e.*, the same geometrical pieces displayed by the model of Brasilia. Model Shop software was used in creating a three-dimensional view of the model which allowed the participants to view any image as a whole or in a

particular aspect, in different angles and levels. Thus, the model could be viewed at different positions and orientations. During the activities I showed the model in different perspectives in order to find out whether the way elements were presented on the screen would make any difference in the participants' interpretations.

Therefore, the setting related to the model was divided into four subsets loaded on the screen. The first one (see figure below) referred to a subset of figures displayed in two-dimensional mode.

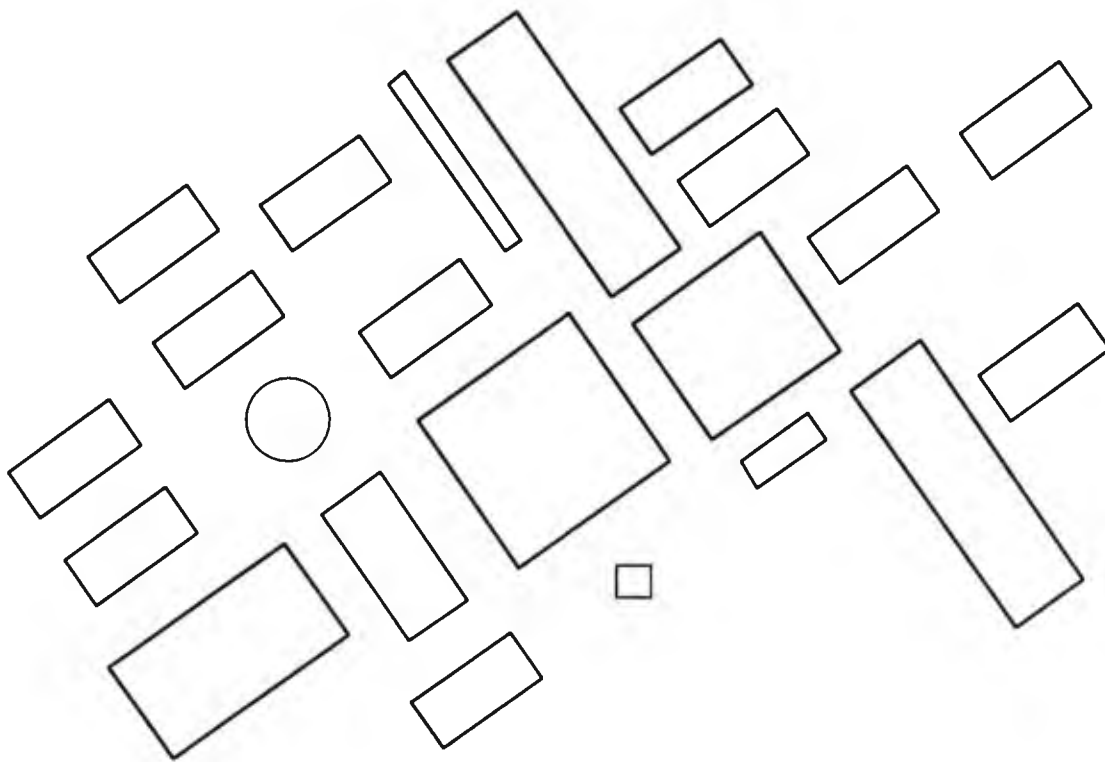


Figure 3.3 A plane view of the model of Brasilia, elements displaced

The arrangement of figure 3.3 was not the same as it appears in the real setting; some elements which composed the blocks of the model were displaced. In particular, the shapes/forms which represented the Department of Foreign Affairs and Planalto Palace were displaced, as well as the base, and the pole of the

flagpole. The remaining blocks were only displayed as shown in the following figure.

The second subset (see below) was the Model of Brasilia represented by a diagram in which the figures equivalent to buildings' shapes were arranged in the same way as in the real setting.

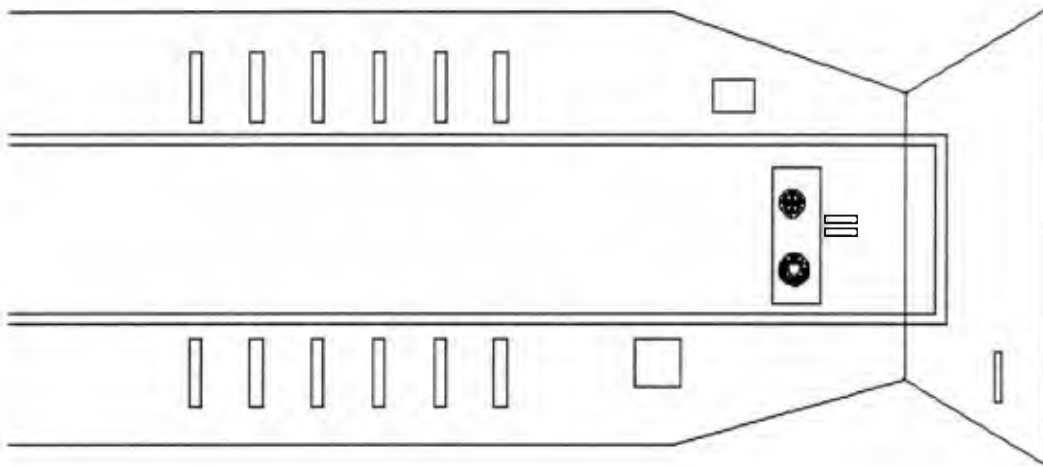


Figure 3.4 A plane view of the model of Brasilia.

The third one seen by participants refers to the model of Brasilia displayed from three different angles. The first view of the model was shown from a frontal plane (see below).

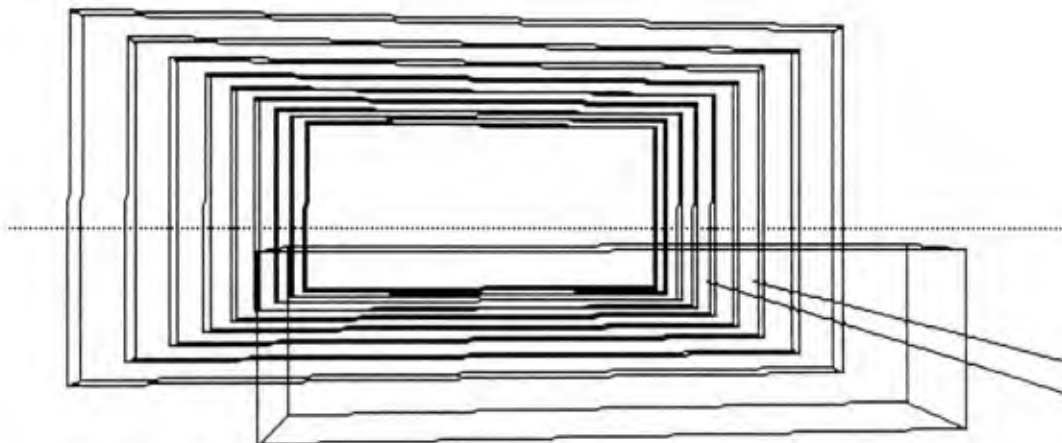


Figure 3.5 A frontal view of the blocks

The other two views were shown from an elevation plane as the figures 3.6 and 3.7 below illustrate.



Figure 3.6 An oblique view of the blocks

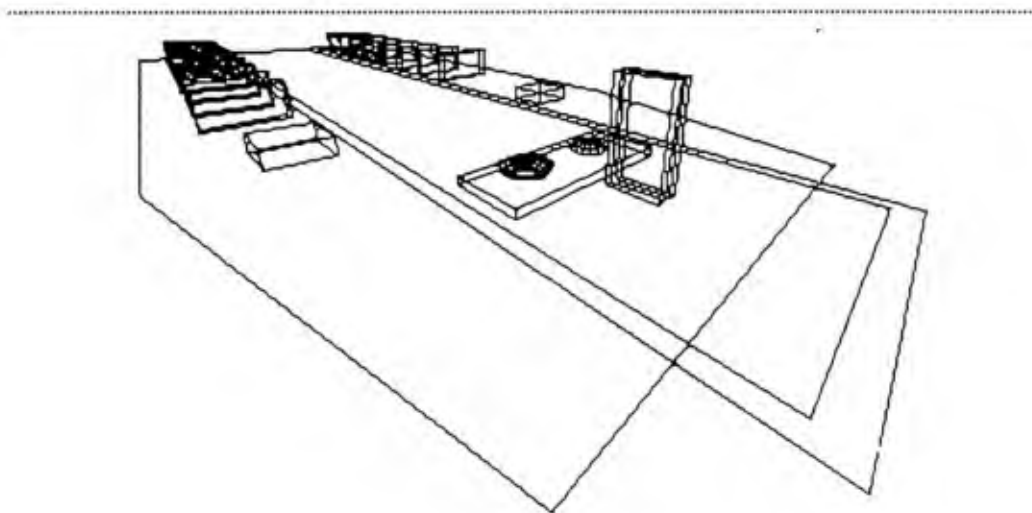


Figure 3.7 An elevation view of the model of Brasilia

The last setting presented to the participants contained a set of sixteen three-dimensional figures viewed from different perspectives. Mac Draw software was utilised to construct the figures and I encouraged participants to enlarge, reduce, move, and spin the figures. All of these possibilities were used during the activity. The activity was not designed to create shapes but was planned so that the participant could manipulate figures in order to increase interpretative choices. However, since some interviewees wanted to construct their own figures, it was necessary to give a quick introduction to Mac Draw.

The geometrical figures shown to the participants were not numbered as they are below. I do it now in order to facilitate the examination of the participants' responses.

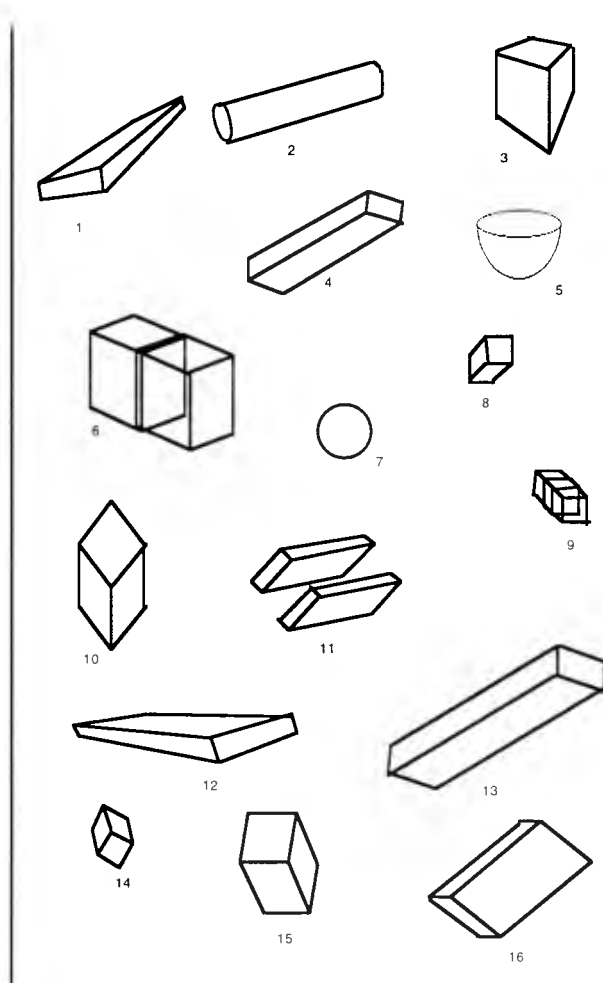


Figure 3.8 Sixteen figures viewed from different perspectives

3.4 The research sessions and the questions explored

Three separate interview sessions were conducted, each lasting more than two hours. During these sessions, the participants produced two types of utterances: verbal and visual - all of which were documented. In what follows, each session will be described in turn.

3.4.1 First session

The first session involved firstly discussions about the model of Brasilia and then discussions about the wooden blocks. The following questions were asked:

Model

1. What do you see?
2. How would you describe it?
3. If you had to describe the model to a friend who does not know what it is, what would you say about it.

Wooden blocks

1. What do you see?
2. What you can say about it or how might you describe it?
3. Can you describe any relation between these blocks and those blocks in the model? If there is, what is it?
4. What are the names by which you know these objects?
5. From where do you recognise these blocks? [In the case that the interviewee might answer that she/he has seen it at school, I asked the following questions, allowing a discursive answer].
 - 5.1. In what discipline?
 - 5.2. What have you learnt about these blocks?
 - 5.3. Do you recognise these blocks in your home environment? At your work? Can you identify them from somewhere else?

3.4.2 Second session

In the second session, a week later, involved firstly discussions about the wooden blocks portrayed in a photograph and then discussions about the blocks presented in the various computer settings. The questions explored were:

Photograph of the blocks

1. What do you see?
2. What you can say about it or how might you describe it?
3. Can you describe any relation between these blocks in the photograph and those wooden blocks? If there is, what is it?

Computer setting

In order to organise the data, I divided the settings shown on the computer into four parts, called activities 1 to 4.

Activity 1

This activity comprised a set plan of geometrical figures which represented the model of Brasilia. Looking at the figure loaded on the screen, I enquired:

1. What do you see?
2. What you can say about it or how might you describe it?
3. Can you describe any relation between these figures and those blocks viewed at the model? If there is, what is it?

Activity 2

This activity was focused on a diagrammatic representation of the model of Brasilia. The arrangement was the same as it appeared in the real-life area of Brasilia. Showing the model on the computer screen, I asked:

1. What do you see?
2. What you can say about it or how might you describe it?

Activity 3

In this activity the model of Brasilia was observed at various moments as the model was viewed from different angles. The area focused on was the same used in the first context (Model of Brasilia) and so the same geometrical pieces were used in this present model.

First step: the screen showed an isolated component of the model, and then asked:

1. What do you see?
2. Do you recognise this figure as part of the model of Brasilia? If the answer to this question was 'no', I showed other parts - in different perspectives- asking what she/he was seeing until they identified the *Plaza of the Esplanade* .

Activity 4

In this activity a set of three-dimensional figures was displayed on the screen. Here I reminded the interviewees that it would be possible to see the figures in different positions, rotating as well as enlarging them.

1. What do you see?
2. What you can say about it or how might you describe it?

3.4.3 Third Session

The third session, again a week later, involved free drawing of geometrical elements shown in the previous sessions. The intention of these drawings was to observe how the geometrical figures and objects would be conceptualised through graphic representation. Their drawings will be used as illustrations throughout different chapters of analysis of data.

In this session, I talked about the city of Brasilia in an informal way and explored questions related to participants' education background and aspects of their life at school. Those who worked were asked how and why did they come to be in the trade and to describe what they actually did in their job. I took the opportunity, at this time, to ask two direct questions about their views on mathematics, and in particular on geometry: 'What is geometry?', and 'What do you know about it?'. Finally, I discussed my research purpose with each person.

3.5 Initial considerations in organising and analysing the data

The task of reading and interpreting the collected data was a lengthy process that underwent a series of stages. During the organisation and analysis of the data, I faced some challenging prospects. The first one, after having the tapes transcribed, was to translate into English interviews conducted in Portuguese, especially the non-structured information collected. In relation to the process of transcription of the data collected, recorded in tapes, I tried to maintain the

colloquial style of the interviews in the participants' speech. This way, in the translation process, I was less worried with putting their speech into standard English than with what was originally meant to be said by the students and manual workers.

Another difficulty I faced was the effort of categorising the data collected. At the initial stage of my analysis, I attempted to place the argumentation - which different groups predominantly constructed - into categories of 'abstract' and 'concrete'. Influenced by Luria's work (1976), I referred to as 'abstract' when objects or sets were characterised by geometrical terminology (cuboids, hemispheres, cylinders). In contrast, the category 'concrete' regarded to attributes based on 'concrete and object-oriented names' in which everyday references were emphasised.

I believed that it was possible to fit the data into these two categories; separating the group's response from each other. However, as soon as I started to examine the data in greater depth, I found that this strategy of analysis was problematic since the boundaries between 'concrete' and 'abstract' arguments were not at all clear. Indeed, the more closely I looked at the data, the more apparent it became that the strategies utilised by different groups of people were not fixed neither was there a single discourse in operation. For instance, the interviews revealed that the secondary students sometimes related the shapes/forms with everyday objects as well as some manual workers and PROEM's students used, although much less frequently, geometrical terminology while they were describing the objects.

At this stage, I noticed that the dichotomy 'concrete' versus 'abstract' created serious obstacles. First, I was not gaining any new insights nor deepening my understanding of the relationship between the geometrical knowledge expressed by different people and their specific social experience; and it was in this part of the research process that these connections had to be made explicitly. Second, in developing this kind of category I was focusing on the 'vocabulary' as if

it characterised their responses. Looking back on the information collected, I realised that this strategy of analysis obscured the complexities of different geometrical experiences since it did not examine the ways in which the different group of persons explored the concepts - knowledge and understanding - which cannot simply be confused with the terminology used by people to articulate what they see - and their relationship to them in which the participants consulted his/her personal system of beliefs and values.

Afterwards, in an additional step, another type of difficulty came up, related to the way I was organising the data. When I started to write the first version of the analysis chapter, the intention was to divide it in three sessions. Each session would correspond to one specific group's response to the four settings explored during the interviews. This seemed the most appropriate way to show the principal findings of my research. However, while I was involved in the process of writing, an important inconsistency emerged: I was isolating the responses of each group without showing any contrast or similarities between the groups. This strategy of organising the data described the utterances as 'monologues' - completely isolated from one another. Indeed, in separating the three groups, I reduced the possibility of seeing variable relations between them. That is, while the groups of secondary students and manual workers may be similar in one particular, for example, they may be different in another.

Both the ways in which I was conceptualising geometrical experience and the way of organising the data were somewhat incoherent and inconsistent to my theoretical position. The contradictions between my commitment to a 'dialogized' analysis; my aim of rethinking geometrical experience in a way which accounts for both their and my explanations of that experience and the relation between the two; and my actual use of clear-cut opposition or well-defined border between 'concrete' and 'abstract', sent me back to my theoretical position. I realised that the issue - of how different people conceptualise visual texts (images and objects) - I was identifying could and should be understood as primarily

antagonist and conflictual inherent in the underlying social relations. Therefore, as the analysis proceeded I tried to understand how the interpretations varied and why there were conflicting interpretations. In the process of analysis I refined and reshaped my initial questions, trying to examine in greater depth the major themes or issues which emerged from the data and the ways in which these related to the focus of the research. The concepts and questions that are central in the final report are different from those with which I started.

In the following section, I shall examine the questions and concepts that emerged during the process of constructing a new analytical framework of understanding. In doing this, my intention is to elucidate my theoretical space and to show how it is related to the empirical data.

3.6 The articulation of the analysis with the theoretical space

In the process of analysing the empirical data I became aware that no absolute separation between theory and practice was possible. In fact, theory and practice, as Newton (1992:3) pointed out, "continually interact and are mutually dependent on each other". In what follows, I shall examine how these terms were integrated in my study. However, in order to understand this integration it is necessary to clarify some points. First, practice, as I see it, is not a mere application of theory. Practice, as reminds Felman (1987:11), "is a process, not a set of doctrines. In the process, one can implicate the doctrines, one can perhaps imply them, not apply them". Therefore, in practice, one can use theories (as I am here to use a socio-semiotical approach) not as a formal set of concepts but rather as a device which will help me to illuminate a way of reading.

In my view, two important assumptions underlie the above statements. First, the process of analysis of an utterance or a text - which constitutes a practice of reading - "cannot be direct, intuitive; it is constitutively mediated by a hypothesis; it necessities a theory" (Felman 1987:23). Second, the theory is a route of reading and not a mere application of a formal set of concepts objectively

which fit the data (for instance my respondent's words or actions). Thus, it is no longer a question of whether specific data support a theory or of simply testing theory against a set data . As Layder (1993:62) argued: "the adequacy, validity and relevance of theory cannot (...) be understood as a simple appropriation of, or correspondence with, the empirical world". Rather, "the issue is about widening the manner in which (...) theory conceptualises the empirical world and how we theoretically characterise it" (Layder 1993:63). Implicit in this vision is the assumption that we (as readers) have to adjust our own ways of thinking and of operating the theoretical concepts in order to analyse our empirical world. In this sense, reading an utterance or a text is an act of construction; "it is a theoretical construction" (Felman 1987:23).

Within this perspective, the analysis of the material collected in the empirical part of my study was the outcome of my reading. Nevertheless, what I encountered within the discourse of the other was not 'discovered' in a direct and instinctive way (Felman:1987). Rather it was mediated by a particular mode of seeing. More specifically, I have chosen a socio-semiotical approach as a major 'route of reading' to understand how different people conceptualise visual texts.

In the following section, I will explore how the development of categories of analysis of the empirical data was guided by the theoretical categories, that is, the analytical tools, adopted in my research.

3.7 The development of the categories of analysis

In examining the analytical tools incorporated in my research - concepts like 'sign-functions', 'sign', 'visual text', and 'heteroglossia' - it is possible to say there is an important convergence. That is, these concepts have in common the question of conflict, ambiguity, and discrepancy. As the main point of reference and guideline in approaching my data was those theoretical categories, it ultimately encouraged me to discard the security and stability of a binary logic of

opposition. I started to pursue a direction where diversity of types and varieties of discourse emerged.

Indeed, concepts like 'sign-functions', 'sign', 'visual text', and 'heteroglossia', helped me to recognise that when faced with the need to analyse any text/utterance for whatever reason, we have, in the first instance, to engage with four basic assumptions:

- (i) that a text/utterance has not a single and fixed meaning
- (ii) that a text/utterance is not itself a 'stimulus' which can be objectively identified and quantified
- (iii) that a text/utterance is not an individual construction
- (iv) that a text/utterance cannot be understood outside its context

These four assumptions formed the premise for the development and refinement of my categories of analysis. First, I realised that it was possible to construct an analysis based on plurality instead of seeking a pattern of clear-cut opposition. With this possibility, I chose a new way to read my data. The first step in this process was to re-map the interpretations and the types of strategies utilised by different people.

My concern was not to search a well-defined border of equivalencies or oppositions between the groups as if each of them had an unique and particular view. Rather, my interest was to locate the diversities, discrepancies, and ambiguity *between* the groups and *within* each group. I aimed to thus identify the general units of meaning, that is, patterns of argumentation and strategies which recurred frequently during the interviews.

At this point, a return to my theoretical considerations may be helpful in clarifying what the categories of analysis - as general units of meaning - represented. As mentioned earlier, analysis, from the point of view of socio-semiotical, requires a breaking away from the idea that meaning is located in the interior of a text/utterance so that the act of analysing involves finding and extracting meaning from something inherent in it.

Motivated by concepts and theoretical ideas drawn from Bakhtin, Eco and Kress, I became aware that in reading the data, I had to go beyond the words or

sentences produced by the interviewees since meaning did not lie in the participants' speech as an act that ends in itself. Therefore, while examining the utterances I attempted to decipher and reflect upon the signifieds underlining in their responses. At this stage, I began to question: what conditions motivated their responses? Searching for this, I had to move constantly from the theory to the information obtained, inferring about the meaning(s) of the data while trying at the same time to catch, translate and express possible multiple signifieds that one enunciation could comprise.

It was within this momentum that I realised the examination of 'words or things' or units of words, does not simply scrutinise sentences, statements on things and their referential meaning. Rather, as stressed by Bakhtin (1981), 'we are speaking about specific points of view in the world, forms of conceptualising the worlds in word, specific worlds view, each characterised by its own object, meanings and values'.

3.8 The Categories

An intense study of the empirical data revealed that a plurality of messages was produced by different viewers and in different contexts. In the process of finding particular patterns two broad categories were distinguished:

- (i) descriptive categories: 'what was seen' in the settings: interpretations and descriptions made by participants
- (ii) relational categories: 'how it was seen' in the settings: relation (ships) identified in and between the shapes/forms of the settings.

Descriptive Categories

'What was seen' through the settings: interpretations and descriptions made by the participants.

I tried to examine discourses which characterised and exemplified geometrical experiences that can be broadly represented by five different dimensions:

Dimension 1- The character of the description of the form

- 1) **Formal Aspects:** Description of the shapes/forms as a formal mathematical construction: using terminology of either plane or spatial geometry.
- 2) **Association with Everyday Objects or Familiar Situations:** Mention of everyday life objects or familiar situations to describe the shapes/forms present in the setting(s).
- 3) **Decomposition:** Reference to answer(s) that describe the shapes/forms where the object(s) or figure (s) was(were) cut or divided into cross-sections.
- 4) **Constructive Description:** Description of the shapes/forms in which the participant tries to explain it as if they were manipulating or shaping the object described.

Dimension 2 - Type of reference

- 5) **Subjective reference:** Assignment of value attributes (powerful; beautiful; good; bad; etc.) to persons or things perceived in the setting.
- 6) **Objective reference:** Any answer that suggested indifference or difficulty to express subjective ideas or feelings.

Dimension 3 - Presence or absence of movement

- 7) **Dynamic:** Any mention of movement: rotating; rolling; moving; walking; flying; etc.
- 8) **Static:** Any answer in which the participants describe the shapes/forms as if they were looking at them from a fixed stationary point.

Dimension 4 - Presence/absence of other characteristic beside form

9) **Appearance:** The shapes/forms of the settings are described by their external aspects: material; colour; texture; ornamental detail; light; dimension; weight; position; style; surroundings.

Dimension 5 - Presence/absence of functional aspects

10) **Functional aspects:** Reference to any utility of the shapes/forms contained in the setting.

Relational Categories

'How it was seen' through the settings: relation (ships) identified in and between the shapes/forms of the settings

1) **Correspondence:** Parallelism between two or more forms/shapes or part(s) of them which implies substitution of a term to a contiguous one or to an equivalent. This is identified by the existence of connectors such as 'like'; 'as if'; 'such as'; 'it reminds me'; 'it seems like'; etc.

2) **Contrast:** Relation of difference and/or disparity between two or more shapes/forms described.

3) **Polarity:** Diametrical opposition between two or more shapes/forms described. It differs from contrast by having a marker of polarisation in the description, identified through conjunctions or adverbs such as completely; only; definitely; not at all; etc. (A is [completely] opposite to B)

4) **Symmetry:** Description of the shapes/forms in which the participant divided it into two or more identical parts and these parts were systematically disposed in relation to one another.

4 The background of the participants in the empirical study

Throughout my investigation of geometrical understanding, I have taken the position that persons do not have raw experiences of geometrical objects: meaning is derived from background, experiences, and social motivation. Therefore, to understand how persons negotiated meaning within different 'visual situations' all of which involved the construction and interpretation of geometrical ideas, we must seek identify the social resources they have available since it is these which provide the basis for the geometrical understanding.

In attempting to locate those social resources, I shall provide, in this chapter, an overview of social and educational background of the three groups of persons in which the school knowledge and practical knowledge will be the focus of attention. To that end, I explore how the two schools where the students interviewed were enrolled have been dealing with the mathematics/geometrical learning process; and I verify the nature of the manual workers' occupation in order to associate their ways of dealing with geometry.

4.1 A formal school

This section presents a general view of mathematics/geometry in a public secondary school of Brasilia. I consider the curriculum proposal as elaborated by the Educational State Secretary for mathematics education in the Federal District of Brasilia (GDF:1982a); and the responses of the geometry teacher who selected the students to be interviewed; observations made by students themselves. My intention is to examine the nature of the Mathematics curriculum through two questions: (i) what is the official justification and purpose of mathematics?¹;

¹ The official curriculum proposal is directed for Mathematics in general, without particularising the objectives for different subareas of Mathematics.

and, (ii) how are the values and needs proposed by the official discourse understood both by the mathematics teacher and by the students.

According to the official discourse - as interpreted from State curriculum proposals, the main objective of the teaching-learning process of Mathematics is:

- (i) to develop logical and critical thinking; to be able to take decisions and inter-relate facts;
- (ii) to organise data and to work out problems, developing the student's consciousness of his/her own rights and duties for the citizenship" [free translation; see original] (GDF 1986b:3:)².

More specifically, the curriculum specifies the purposes of mathematics:

to correlate; analyse; assess data and results observed; synthesise; abstract and generalise problem-situations; develop reading habits and rigour and precision, order and clearness, of correct language using, critical reasoning and discussion of results; and to develop skills of expressing and comparing measures; recognise and set spatial relations; calculate; draw and interpret graphics; recognise different fields of the mathematics domain and of other sciences in which physical laws and intellectual approaches are developed from the mathematical research (GDF 1986a:30).

Mathematics education at the secondary level aims at developing skills and competencies. Its purpose is to give:

continuity to the universal changing from a fragmentary view to a semi-continuous one, in which the knowledge applications are not necessarily to the short term, but one that permit the learner, along his life, an approach to the scientific knowledge (GDF 1986a:30)³.

According to this viewpoint, while the learner deepens and broadens knowledge, she/he also makes possible the use of the scientific method, of logical thought and develops a critical mind (GDF 1986a:30)⁴.

The geometry's teacher interviewed (Clovis) however, identified a problem with the objectives of this proposal saying:

² O ensino-aprendizagem da matemática visa o desenvolvimento do pensamento lógico e crítico, a capacidade de tomar decisões, de relacionar fatos, de organizar dados, de equacionar problemas, tornando o aluno um elemento consciente de seus direitos e deveres como cidadão.

³ "O ensino de Matemática no 2º grau visa dar continuidade a sistematização do saber universal, passando de uma visão fragmentada para uma visão semi-contínua, em que as aplicações não sejam necessariamente, a curto prazo, mas que permitam, ao longo da vida do educando, um acesso ao conhecimento científico" (GDF 1986a:30).

⁴ "O aprofundamento e a ampliação dos conhecimentos possibilitam o uso do método científico, o exercício do pensamento lógico e o desenvolvimento do espírito crítico" (GDF1986a:30).

The educational aims and content are elaborated a priori, and the teacher shall accomplish them in order to fulfil the Secretary of Education's aims. In my opinion, however, the teacher should establish aims accordingly to the students' knowledge level. What the school does is to propose a method of curriculum assessment and evaluation, and, we, as teachers, adapt ourselves more or less to get near to the particularities of our classes.

What is significant is that the official discourse cannot take into account the specific engagement students might make with mathematics. That is, while the State model presents mathematics from a logical-analytical model, the teacher seems to be saying that what matters most to his pedagogy is a consideration of the socio-cultural context of students. Thus for the teacher, improvisation and adapting to the students are key processes.

The five secondary students selected for this research project were chosen by their geometry teacher (Clovis, cited above) as representing those with a high level in geometry. According to him, "they are the best ... the top level and all of them would not have any problem in entering to the University". Among the students chosen, two were considered by Clovis as:

very good in memorising formulae ... in this sense I could call them as traditional students while the others three are more sophisticated since they are more reflective and thus they solve problems more easily since they are more concerning to understand what the problem ask rather than memorise formulae.

Because the main priority of school is to prepare students to enter university, the highly competitive classificatory exam (called "Vestibular") has significant implications for the design of curriculum and instruction. So, an important aspect of the pedagogical praxis for secondary school teachers is timing. According to the teacher Clovis this is indeed the most important element in putting the educational board's aims into practice. As he stated:

Teachers are not free to talk about questions raised or problems brought out by students because they have too many content to transmit and too a short time; therefore, unfortunately, it is not possible to give a closer attention to each student and we have to ignore the fact that each student has his/her own learning individualities.

What this brief extract reveals is that the curriculum not only defines what shall be taught and what knowledge is, but also defines how knowledge is taught. The curriculum involves a set of relationships between the teacher and the student.

But as this teacher points out, the curriculum, more often than not, prevents teachers and students from critically engaging with their own questions. This problem is articulated as one of time and timing. According to the teacher, besides the constraint of timing:

The relationship between a teacher and a student becomes worse if we try to balance the great number of students per class and the extension of the program. This makes it necessary for us to give lectures and to talk continuously, and even worse, rapidly.

The pressure of time is also a permanent concern among students. Here is Alex's analysis:

The other day I was making a calculation about the time I spend between home and school; I waste two hours of my day time inside the bus. And I was taking as a basis if we have 180 days of the school calendar, thus I waste 360 hours per year inside the bus. If I lived near school, and taking into account as my average of studying to be 10 hours a day, I would have an extra 36 days to prepare for the 'Vestibular'.

Ana, another student observed:

As you know ... we are being trained to take the Vestibular in which we have to state the problems quickly, state exactly what is given and to give the correct answer. As there are many questions to solve we have to work rapidly.

This pressure of time, as it was observed by the teacher and by some students, is significant for the process learning of geometry. More specifically, geometrical content is generally concentrated in the final part of the books and, since the time available to study is often not enough to cover the whole curriculum with the result that a great part of geometry is missed. Besides this 'normal' constraint, as the student Carlos noted, over the previous two years, classes had been interrupted many times due to teachers' strikes. Hence, the teachers had to accelerate their curriculum delivery even further, causing more compression of the topics. Another student, Elisa discussed the pressure of time in relation to the length of lessons, currently forty minutes. In her opinion, "they should be double because the teachers don't get enough time to explain things". According to Elisa, some teachers require silence, - "very soft whispering", and there are others who require that the students raise their hands to receive permission to speak. This comment was accompanied with the view that: "usually the teachers speak

most of the time but when they are in the middle of their explanations they have to stop".

What is significant in these students' observations is their understanding of the difference between the time given to the mathematics curriculum and the time they need for their own learning. This is a qualitative problem in that the actual conditions for learning geometry are felt from many vantage points not to be adequate to develop a rich geometric understanding.

Indeed, in reading the above comments made by the geometry teacher and students it is observable that neither subjectivity nor negotiation are part of the students' classroom experience. In this traditional context, the relationship between teacher and student is hierarchical. The teacher is seen as someone who possesses knowledge and whose task is to pass it on to the learner in his/her charges. The learner is expected to acquire the body of knowledge transmitted and to apply the rules underlying it. Therefore, there is no space to recognise differences in the way students explore concepts and relate to them. This constraint, as we will see in the next chapter, is well understood by those students who have managed to think critically about geometric design.

4.2 An 'alternative school'

The PROEM ("Educative Promotion of the Under Aged") school, localised in the central part of Brasilia, is the result of an alternative project supported by the local pedagogical board of the Secretary of Education in the Federal District, Brasilia. It is organised by a group of social workers, teachers and psychologists, all with strong pedagogical orientations. They work with students who, as children and/or teenagers, abandoned school. The students usually live as under-employed persons, pickpockets or street children until they come to PROEM. One of the most important aims of PROEM is to offer a substitute for formal education by means of an experimental experience, adopting student-centred approaches, innovative pedagogical practices which a flexible curriculum. There are some

preconditions that persons to be admitted: to be working in the formal or informal market; to be at an age level too high for the school grade; and to be referred by a social worker organisation. The educational level offered is that of elementary school where the prior aims are to develop general aspects of students' cognitions, self-esteem, creativity, and social skills for use in school and in the local community (Huthmacher:1988).

According to Vivian, a Proem's mathematics and computer teacher,

the teacher is after all a friend, concerned above all with the good relationship between teachers and students, encouraging students to like school, like maths classes and like their colleagues. Our main role as teachers is to try to prepare the group achieve a better life.

A main educational task of the school is to extend the life skills of pupils by inculcating such qualities as self-confidence, patience, and perseverance which cannot be done in the home. Part of the educational value of the school lies in the practical opportunities provided for learning to mix socially and to communicate with many different kinds of people.

The teacher Vivian selected five students for research participation. According to her, two of them, Gil and Maria, have "a language pattern that is very poor, and a low level of general understanding ... finding difficulty in comparison and generalisation that math studies demand". She considered the student Antonio "the most intelligent one and the most logically-minded". As for Andrea, "she is a teenager poorly stimulated, being slow in any kind of problem-solving".

By reading the school's proposal and the interview with the teacher, I could see that the professionals of the institution assigned the low outcomes of the students to 'emotional problems' arise from the family environment: 'an alcoholic father; a mother out at work; separated parents'. Or, school failure is ascribed to individual 'lack of skill', 'lack of motivation' or even to environmental problems. The teacher interviewed, however, mentioned neither the structure of formal schools and social structure nor the living conditions of the students.

Some students considered, as the teacher did, their inadequacy for studying as being their own responsibility. For instance, the student Gil has enrolled at the third grade of the primary level. He repeated the first grade for three consecutive years and the second grade once. For Gil, the task of studying is not easy, as he said "sometimes I like to study other times I don't. I have many difficulties and I believe that my memory is weak". Recently Gil has sought to recover the inadequacy for the studies in which he considered to be his own failure. He explained this in the following words:

I didn't make enough effort for it, now I want to make up the time so lately I have become different. Before I did not study anything and all I like was to cause confusion and disorder.

The student Andrea, like Gil, considered her failures to be her own responsibility in that "if I had worked hard I wouldn't have failed, but my mind was very dull in which I couldn't memorise anything". On the other hand, she blamed the teacher's attitude as an important factor that influenced her lack of motivation to study.

Thus she observed:

The teachers from my previous school didn't give any stimulus. On the contrary, they didn't have any patience with us, i.e. if I didn't understand a topic I asked them about it, however, they used to say 'I'm not going to repeat it again because there are many students so I can't repeat every one of you doesn't understand'. Over here, the teachers are much better because they repeat even 10 times if it is necessary.

In contrast to other students, Maria believed her failure to be due to the poverty as the below extract illustrate:

I'm not good at school ... I repeated the first grade three times and in the second one twice ... I think it is poverty that caused my slowness. All of us are too poor to buy some basic needs like fish, meat or milk that contain vitamins. All of us have this deficiency and maybe this is the reason for our problem.

This observation made by the student Maria, contrasts with the teacher Vivian since the poverty factor was never mentioned by her. Rather, the teacher emphasised that without 'positive attitudes' and 'discipline' students have little chance to learn proficiently. This is explicit when she talked about maths:

Mathematics is the science of abstract and precise reasoning. But it is very difficult to teach this to the students who come to this school because they have low stimulation at home and many of them have learning difficulties.

She went on:

I think that the self-discipline is fundamental for improving academic achievement. Unfortunately in our school we are confronted with students who don't have any self-control.

Yet, the teacher Vivian believes that deductive reasoning as required by the formal mathematics curriculum is beyond the capacity of the PROEM students who "don't have clarity and accuracy of thought ... rather, most of them are impulsive and have emotional problems". Thus, "they are unable to cope with the abstraction and generalisation demanded by mathematical tasks". She concluded saying: "facing this kind of student ... our priority cannot not simple acquisition of knowledge but rather our main goal is to try to foster positive attitudes about learning".

Explicit in the above observations, is that formal academic learning takes second place to teaching skills in everyday social living. therefore, the main aim of school is to give a minimum 'civilization', as observed by the teacher: 'it will enable the students to have a sense of belonging in society'. By analysing many commentaries made by the students, a strong presence of these social values is apparent when they talked about their experiences at PROEM's school. For example, according to the student Andrea, the most positive aspect of the PROEM is the motivation developed by the teachers in their pupils. As she explained: "Here we learn about life, I mean they teach us to face the problems of life and help us to behave better. I have learned many things here, especially about friendliness and how to live honestly". With regarding to the process of teaching time was not such a constraint as Andrea noted:

The teacher wants to put the things inside our heads and for this they have patience, and also they know a poor person doesn't learn in the same rhythm as a rich boy or girl. We usually arrive here full of problems especially because we had failure in the normal school. Therefore, the teachers at the PROEM demonstrate the things more clearly and there is not such pressure as at a normal school. I mean, when we do not understand what she is saying she sometimes shows through objects, i.e.; she shows six apples in order to teach if we take away two from that amount there that will leave four. This is demonstrated using real apples.

Kelvin, as Andrea, compared the PROEM with the formal school from which he was expelled: "from the first moment I entered at that school I liked it. The teachers are very nice people, I like them. They treat us as if we were their sons."

In relation to their performance in teaching content Kelvin remarked that "some teachers know how to explain others don't; when they have patience and clarify things, I don't have problems in understanding". However, this last aspect was not that important. In his opinion, the experience at that school was more significant in terms of acquiring good attitudes towards life rather than the knowledge delivered in lectures. He thus observed: "when I entered here my self-esteem was very low. I didn't have interest in anything, it's like I was living but I wasn't aware of it". A similar point of view was mentioned by Maria and Andrea who repeated the first and the second grade many times,

maybe four or five ... to be honest with you I lost count how many times. I like to study at the PROEM because the teachers treat us like their daughters or sons, so they give attention and affection. In this manner, it is very different from those who always humiliated us by their disdainful remarks. They always reminds us of our poverty, saying that we never could have a better life because we are poor and stupid. So, who is poor dies poor.

At the PROEM I met various boys and girls, some good, others bad, but the most important is that I learned to deal with all of them and nowadays I know what is good for me. Another good thing over here is that the teachers treat us as human beings. It was important to me because I gained confidence in myself.

Antonio, like the other students, commented about the formal school from where all of them were expelled, and emphasised the 'patience and friendliness' of the PROEM's teachers:

I had repeated twice the third grade in another school where I rarely attended the classes. I didn't like to conjugate the verbs in the Portuguese lectures or to memorise multiplication tables because my mind didn't work well in memorising it. I usually fell asleep during the classes and when it happened the teacher used to wake me up screaming in my ears, I hated those moments. I prefer the PROEM because the teachers treat us as humans and not like animals.

In relation to the knowledge acquired at PROEM, Antonio stated: "Of course, I couldn't say whether it was useful but it seems to me I haven't learned much about mathematics or Portuguese, I wish I could have learned more about it" [my emphasis].

For me the above words revealed an intense hope for a better life, a desire for change and better opportunities. In reality, Antonio's life is not remarkably different from others like him who have lived a deprived childhood, in a social reality marked by a world of violence, of deprivation and suffering, of an immense inequality in income and an extreme low purchasing power of money. Indeed, as the student Gil noticed,

The people outside consider that all of us are marginal and vagrant, and it is not true. There are some who really steal and burgle but it is not fair to say that all of us are thieves; it is a lie though this is the price that we pay for being broke and poor. I would like to find a job so I could buy some clothes and I would send money to my mother who has had a miserable life. But I don't want to find a job as a servant who support the weight of the bricks and takes it from place to place. For not being like this, I have to study.

The student Maria agreed with Gil:

Although we are competent and good workers, devoting ourselves to work with our body and soul, the majority of the population in Brasilia imagine that the boys and girls who study at the PROEM are abandoned and marginal.

Recently, she has been anxious about the possibility of losing her job. What worries her is the difficulty of finding other work in the present economic situation. Moreover, she is of the opinion that the fact of being a student at the PROEM makes things harder for her, and if she does not give up studying it is because her mother always says: "you must study if you don't want to be like me who has passed my whole life in the others' kitchens. When I listen to my mother saying that I lose the courage to get out of school".

There is an ironic contradiction in thinking about the dilemmas of these students' lives. It has to do with how living in poverty inhibits educational success or even the thought that education can matter. The other problem is that education itself sets this impossibility in motion.

4.3 The manual workers

In order to obtain access to search out the root of manual workers' geometrical knowledge, from where that knowledge originated, it is necessary to examine the nature of the work; and to access the role school had in structuring that

knowledge. In attempting to tease out these influences, I shall begin by saying that all the workers' histories which will be presented here have one point in common: like millions of children and adolescents in Brazil all of them left school when they were between ten and fourteen years old. As a product of the same exploited social class all of them had to leave school either in order to support the family financially or for the reason that their parents considered the school as a place for the wealthy families. Thus some of them dropped out of school in order to have a trade.

The carpenter Sebastian came from a state situated in the North-East of Brazil, one of the poorest areas of the country. Until the age of ten, ten years ago, Sebastian worked on the land and attended a rural school. After this period he did not receive any formal knowledge education.

His career as a carpenter started in his father's work shop:

I grew up in my father's work shop who was a self-taught artisan. There was a fair amount of antique restoration carried on there and I learned to know how to hand and cut wood ... my father taught me while he was working. When I was ten years old my father started to give me some works. At this age he told me that it was time to get out of school and to dedicate all my time to working in his work shop ... my father had the opinion that school was a waste of time ... he used to say that his sons should have a trade and not to learn theories taught at school which are worthless.

By observing what was said by the carpenter Sebastian, we can notice one important aspect which marks his trade knowledge. That is, his observation 'my father taught me while he was working' indicates that the knowledge was transmitted from father to son through apprenticeship without resorting to any theory or school. The process of learning was established throughout everyday work.

The trade path followed by the carpenter Raymond was not substantially different from Sebastian. Raymond was destined by his mother to become a carpenter. As he observed:

My mother held that all of us had to have a trade, as she always said 'the children must be decent' and to have an occupation was considered by her the only way of being decent. In fact, my mother's view about education was very pessimistic, that is, she was of the opinion that education is not a matter of importance to poor people rather it is privilege of rich people.

Thus when Raymond was ten years old he started to learn carpentry in the afternoons while in the morning he was at school. At his work as an apprentice, there was nobody who gave him direct instruction or supervised his work. At the beginning he just observed other people's work. As he noted:

I learned by looking at people working and there was not any tutor who could say what I should do. After spending some months only watching. One day I took a piece of wood and started to carve it. At that time, I liked to make sculpture, faces of people. One day the chief of the carpentry said to me: 'you have very efficient hands' then he started to give me some work.

At that time Raymond didn't receive any salary. He worked as a volunteer until at thirteen he dropped out of school and took a job as a carpenter and was able to support himself and his family by making furniture for a shop, toys for children, and other objects that the neighbours asked him to do.

One year ago he learned from a friend that he might have a better opportunity in Brasilia, working as a sculptor. After eight months of living in Brasilia, Raymond started to work as a tutor in the Museum of Brasilia. By comparing his work experience in his home town (Bahia, a Northeast state) with Brasilia he observed many differences between the formal and informal ways of learning:

In Bahia, although I did not work as a tutor, there were always people who wanted to learn carpentry. Thus when the people appeared at my workshop I just used to say 'OK, if you want to learn it, just come and we will see what happens.

For him, the situation in Brasilia was very distinct as everything was wrapped around in a heavy bureaucracy, thus "I can't say come on, I will teach you. Here in Brasilia there is a process in which one has to enrol at a school organised by the State, having to show documents proving that a guy is a legal citizen". For this reason, Raymond believes that in his home town there are many more artists than in Brasilia, because the process of learning is more spontaneous and flexible. "If one wants to learn, for example, music or painting all she/he has to do is just look for someone who knows it. There is no necessity to enrol in any school nor for any documents and you pay whatever you can afford".

The potter Tom works at the Cebem, a federally funded education program. The program was open to the community with an average attendance of one hundred and forty people of diverse ages, ranging from teenagers to elderly people mainly housewives. They organise tutoring, arts and crafts programmes.

Tom's workshop is composed of various cylindrical tables, a potter's wheel and a kiln. All of them are built by him utilising the material found in the environment. In fact, all of his work tools like the way he works are rather simple. The process of production of the earthen pots begins with the treatment of the clay which passes through three main stages: firstly, the clay is crushed into very small pieces with a hammer, then it is sifted, and finally kneaded. These processes are carried out in bulk while the pots are individually made. The pots are made by hand or using a potter's wheel - a horizontal revolving disc on which wet clay is shaped to make the pots. The last step of the process is to place the items in a kiln in which the pots are fired.

I met the potter David through Tom who knew him as a colleague. David's career as a potter began when he was living in an institution where young offenders were sent to be trained. He told me of this experience in the following words:

At that time I was fifteen, there I stayed more than one year. There were a lot of manual activities and we had to choose one of them in order to be motivated. At the beginning I hated that place and several times I ran away from there. One day I met a woman who was a tutor in the ceramics workshop ... what occurred between us was very important to my life. I started to gain more knowledge from her and I began to be optimistic and energetic.

After six months of learning pottery, David went to Mato Grosso (a Central-West State) to improve on his apprenticeship with a ceramist. According to David, this experience was fundamental for his career. The tutor lived in a small village, which was difficult to reach by road and they were living and working in very primitive conditions. David learned the use of basic material, the potter's wheel, and firing in a simple kiln. The life there was arduous and the training rigorous. The clay had to be constantly dug out and wood chopped for the kiln. As David observed "I used to do nothing but throw pots during an eight-hour working day".

David's school experience was almost nil. He explained this lack saying:

If my parents hadn't been poor I could have gone to school but my destiny was other. My childhood was spent most of the time roaming the streets, running away from the police who used to treat me with a lot of beating and always sent me back to the reformatory, where I hated to be, so I attended normal school for a very short time.

During the period that he was at the reformatory there were several manual activities as well as "some theoretical classes like Portuguese and Mathematics, though I always preferred to be involved with practical activities like working with wood or wicker-work".

The graphic designer Luis again had very little experience in formal education. He was registered in a rural school until the age of ten. According to him, "there was one teacher and only one classroom where children of different ages, and even some illiterate adults attended". There he learned the four operations (addition, subtraction, multiplication, and division), to read and to write. Luis explained the process of learning applied at that school:

First, you had to memorise the alphabet, that is mechanically repeating the ABC's, the same happened with numbers, we were like parrots repeating numbers and words. After this first stage you started to arrange the syllables and then the words. There was a second step which was the primer which you learned from memory so as to be able to repeat the words exactly. At the end of this process you finally read. There was nothing left for learning at that school because the basic reading, writing, and the four maths operations were all that they taught you. Thus I left the school at the age of ten.

Mario's career as a graphic designer, according to him, began in childhood when he played with material found in the environment. As he stated:

When I was in Piauí [a north state of Brazil] I didn't know anything about graphic design but I already had a sort of instinct for this subject. I was very young when I started to draw with red and yellow clays that I found and which I utilised as chalk, and also I used charcoal. I drew or painted various things, such as aeroplanes that I usually saw crossing the region, the plants that I saw in the surroundings, and the goats that grazed all over in the region.

When Mario moved to Brasília, he worked as an apprentice for two years, as a technician in building construction. According to him:

The person who does this type of work is able to understand the material of the construction, to interpret and draw a plan, and he is capable of grasping a project which involves work from the foundation of the building, through the installation of utilities up to the finishing of the building.

However, as he mentioned, the tendency was to specialise in one of these areas. His career as a professional designer continued in the area of cartography in which he specialised in aerial map-making photography. Meanwhile he took various short courses in graphic-design, silk-screen processing, and nowadays he is taking a workshop on graphic design in newspapers. Although he has experienced in various fields, Mario considered himself a technical and graphic designer "who makes architectural design. I draw with a pen, ruler, compass, and protractor. It's objective drawing". As a graphic designer, he has worked at a graphics workshop at the University of Brasilia since 1986. Mario operates in a vast diversity of assignments which he considers to be amateur work instead of being professional as he would wish.

Here the work is very varied. We must have many-sided abilities because it is necessary at the same time to design, to arrange visually and to finish. Even when you don't know the problem you have to find out a solution. Usually we end up by doing the work, it is a question of playing with the things without being fixed.

As I mentioned at the beginning of this chapter, my interest in providing an overview of the participants' social background and experience has been to offer a basis for a better understanding of how they negotiated meaning in different settings explored through the empirical work. In other words, we shall see in further chapters, how the formal and non-formal schooling influenced the 'voices' of the participants in different situations.

5 Abstracting an understanding of geometry

There are two parts of geometry: the theoretical and the practical. The theoretical is that which contemplates proportions, quantities, and their measures by the speculation of the mind alone. The practical is when we measure the unknown quantity of something by the experience of the senses. (Gundissalinus:quoted in Victor 1979:22)

Practical geometry investigates lines and surfaces which are in the wooden body if the person who uses this science is a carpenter, or in an iron body if he is a blacksmith, or in the surface of the earth and fields if he is a surveyor. (Al-Farabi ibid:11)

As Gundissalinus makes clear, there is a difference between the articulation of knowledge as a set of properties and the actual enactment of knowledge as praxis. In this chapter I shall examine how three groups of people who differed radically from each other in relation to their geometrical experience, reacted when asked two direct questions: 'What is geometry?', and 'What do you know about it?'. These questions were introduced at the end of my last interview after the participants had been engaged in a variety of activities through different settings. During these activities I had probed their geometrical understanding in context which encouraged participants to formulate and express their own views about the material presented to them. In contrast, through these direct questions they were asked to abstract an understanding of geometry that descontextualised knowledge and skills from their practical reality since the participants were asked to theorise an understanding of geometry¹. In so doing, I wanted to examine how the participants would react to these two different ways of communicating.

On the whole, the responses of the secondary students could be grouped into two types: some tended to conceive geometry as a set of rules which be practically applied; others pointed to the importance of appropriating strategies when thinking through geometric problems. For example, the student Marcelo believed that solving a mathematical and geometrical task was an act of mastery and control.

¹Although the participants were questioned about geometry in particular, many of them talked about mathematics in attempting to be specific about geometry.

For me mathematics is like playing chess, when you move the pieces in attempting to capture the opponent's king. I mean, in a maths problem you have to capture the answer as if it were a prisoner. However, the way you trace it must be in a logical manner because nothing happens by chance, that is, you have to follow rules.

My reading Marcelo's responses would assume that whilst he upholds the importance of strategy in resolving, by reducing the question of geometry to one of applying rules, his desire to "capture" answers might actually inhibit his geometric imagination.

Elisa, in her turn, believed that there were tactical actions for solving geometrical problems and could describe them. The first step was to observe what was given and what could be stated from it. Then it was important to identify exactly what was being asked and what must be proved. After doing that, it was necessary to make a construction using a formula, and finally prove it, in order to observe if the construction was correct. This process, as Elisa observed, has to be systematised:

It must be organised, there is no possibility of doing everything at the same time, in a disorderly fashion. Rather, you have to follow certain rules which have an order and logic.

Again, rules constitute geometric competence and the learner's work is to apply these logically. The student Ana considered that:

Mathematics is a discipline which develops logical reasoning ... in doing it we must memorise the main rules and formulas. Maybe geometry is a mental game, I'm not sure because I never thought about it before. Let me think ... when I do a geometry test I don't draw the figures on paper in order to solve a problem because I already have the figure in my mind. In this manner, I save time and at the same time I execute the possible analysis of the problem in my mind. Thus we may say that geometry is a kind of mental game in which you must follow some restrictive rules; these rules are fixed. I mean, mathematics cannot be considered only a game since it is in reality a science which has precision and exactness.

Now this suggests a problem underlying the belief that geometry is only understood as a set of rules to be applied. This is that many of these students begin to understand reality itself as fixed and static, something which might be regulated but not constructed.

A similar opinion was stated by Carlos who conceived mathematics as a formal language based on precise rules and forms. For him, geometry was the

study of figures and shapes such as cubes, cones, cylinders and prisms which are treated mathematically through calculus, definition and theorems. Carlos observed: "it is necessary to memorise and understand the formula". When explaining what he meant by "memorise" and "understand", he said:

I don't know how to explain it, but there are precise rules which we have to accept as conventions, and so we have to memorise them in order to solve the problems. Sometimes I get confused with some formulae but most of the time I'm good at memorising.

The student Ana stated that ninety per cent of the students in her class have difficulties in maths, mainly in geometry. According to her, they do not possess the kind of reasoning necessary for learning. Her opinion was that this could be explained by the lack of a mathematical background and the hierarchical nature of mathematics - a deficiency in the basic principles, building blocks from which knowledge accrued. Without this foundation, the students could not make progress in their subsequent mathematics. She remarked that:

When someone does not have a basis, he just isn't able to understand what comes after. It does not make sense to know how to multiply, and still be unable to use addition; for this reason the majority of the students do not follow the topics given by the teachers. Thus they take the formulae and want to apply them; most of the time however they get them wrong. This does not help at all (...) without a basic knowledge, which sustains what comes after, any effort is in vain.

However, this rather naive view of geometry as simply a problem of application was not held by other students, specifically those who criticised the teaching method utilised by the traditional school. For example, according to Elisa, knowledge consisted of the accumulation of facts and isolated information, transforming the process of learning into a "pure process of memorising". She complained of many mathematical exercises which did not stimulate reflection but simply required a formulaic execution without any real understanding of what this meant. In her view, this was a useless way of teaching. "The student learns what 'two plus two' is but does not know what 'two plus three' amounts to. Thus students are not able to make any connections, and "the majority of the teachers, ignoring the 'why' of the questions do not explain the topics". For her, there are only two possible reasons for this behaviour: the teacher's laziness or

their own lack of understanding of the topics. In relation to geometry she noticed that:

It is not possible to deal with spatial geometry if you don't understand the plane. There is no way of separating them because they are very close. However, for the students and even the teachers the approach to teaching the subject is as if they were different and distant.

Elisa did not agree with her geometry teacher who started to teach spatial geometry and volumes without mentioning the intersection of planes and parallel lines. As she understood it, points, lines, and planes were fundamental and basic concepts of three dimensional geometry and, therefore should be the starting points. Another student, Alex, agreed with Elisa's point of view. According to him:

Plane geometry, with its definitions, is the basis of knowledge for the geometry of volume. I mean if one has a good basis in plane geometry ... it will be easier to understand solids since this involves concepts of plane geometry. However, the teachers don't make the link between these two geometries ... they are treated as if they were two things completely different from each other ... I only started to associate the two geometries when I started to study it with a colleague.

In fact, according to Alex, geometry is not well taught at school. From his point of view, "it seems to be that the majority of the maths teachers don't like geometry or don't understand it". Another gap in the geometry lessons, according to Alex, lies in is the treatment of algebra: "the majority of algebra is deduced from plane geometry, however the teachers don't explain this aspect". Whilst appreciating the relation between algebra and geometry, Alex stated that geometry should be treated with "elegance without any 'algebrism' ... the maximum that should be used in geometrical problems is an second grade equation. In my view, geometrical reasoning is enough". Yet, another of Alex's comments concerned the exclusion of descriptive geometry from the curriculum:

In my view, to remove it from the curriculum was absurd. I realised how ridiculous this was when I started to study it for the Ita's exams, the only University which tests Descriptive geometry ... in studying it I saw that it may be helpful for any geometry since it is the graphic solution of points, lines and plane problems in space. When I and a colleague started to study it we had a vague idea of geometrical design ... thus we started to make some connections with plane geometry. After a while I realised how design is fundamental because we see in practice everything that is demonstrated in abstract terms. In this way, we can solve a geometrical plane problem utilising only the design, here it is enough to know the concepts without rules in order to assemble a problem. I

really don't understand the logic of the people who devise the mathematics curriculum.

Lastly, Alex stated his own approach to the study of mathematics: "my method of studying is speculative ... I seek to make demonstrations and to justify - logically - everything. I really like abstraction ... my favourite subject is combinatorics. I like this because it demands reflection". He concluded saying:

You have to understand what is happening because maths is not memorisation or mechanisation. Maths is understandable but you must understand if you want to do it.

Significant is his view of geometry as a conceptual problem of design whereas earlier students saw geometry as something quite stable, built bit by bit.

The student Marcelo considered mathematics to be a science that investigates the dimensions of numbers and of space in an abstract and logical manner. However, he observed:

Usually mathematics taught at school is treated as something very serious and logical in which few students are indeed capable of grasping its logic. In my case, maybe because I have always liked maths, I view and work at it with a certain pleasure. In my opinion, to follow its logic doesn't mean that you have to be rigid when looking at a problem. These rules must be understood otherwise you can't find the solution. The majority of students try to memorise the rules instead of analysing the problem. I don't blame them because the way the maths is taught leads us to think like that.

Marcelo was aware that the mathematics taught at school is concerned only with its abstraction through representation and diagrams and not with physical shapes.

Marcelo noticed:

Geometry taught at school doesn't make this link but I always try to connect what I have seen around me with the content given by the teacher in the classes. In my case, only recently, I would say one year ago I started to be aware of that. I must say that I didn't learn it within the classroom but rather I began to face a maths problem in a different way when I started to study with a colleague who was very good in maths and acted in an incredible manner in relation to the problems. I mean, he used to do a kind of juggling with the formulae. I was sorry that I hadn't realised it before because I spent many years solving maths problems in a rigid, very stupid manner. Thus I have just started to be less stupid.

The above comments reveal considerable variation amongst students' way of conceiving geometry. Most of them believed that competence in geometry meant the ability to apply rules. In this case, they conceived the discipline as an exact science, formal, finished, and mainly dissociated from concrete reality. This is in

line with the curriculum and its implementation by teachers. Some managed to criticise this viewpoint and began to make connections between the two sides of geometry - a science of the plane and space and a logical axiomatic system.

When I asked the PROEM's students the direct questions: 'What is geometry', and 'What do you know about it?', a common reaction was: "hold on, teacher! This isn't a test, is it?; or "Is it an intelligence test?'. When I rephrased the question still using the word 'geometry' their answers were the same: "Geometry?! I don't know what it is ... I never studied it"; "I haven't a clue, what's that? Geometry?!" or "I don't know what it is".

Some students managed to relate geometry with mathematics: "it rings a bell. Has it got anything to do with maths? If its anything do with maths I don't know anything about it". In a similar way, another student said:

Mathematics is the most difficult subject at school ... I really don't understand what the hell the teacher says ... it is a mystery to me. The people who know mathematics must be intelligent to understand it. My maths teacher is a very nice person but unfortunately ... when she is talking about pure mathematics I don't understanding anything ... every multiplication I do I get it wrong ... I try to get it right but it is impossible.

Another student, Kelvin, said that mathematics was a puzzle in which he considered himself ineffective. In his opinion the hardest thing was to be able to divide: "I'm very weak in dividing numbers, from time to time I keep trying to solve a problem for almost half an hour, struggling to discover the solution".

Antonio, who used to serve an apprenticeship with a carpenter, was the only student who was aware of geometry as an explicit domain of mathematics. In explaining what he understood by geometry, Antonio contrasted the geometry taught at school and that manner of working with that used in carpentry.

When we are doing carpentry we work with geometry but the way of doing it is very different from in school. For example, the instruments used are not the same. At school there is the compass while a carpenter uses the T-shaped instrument for making or testing right angles when he is making a shelf for example. However, at the time the carpenter is making a piece he doesn't imagine that he is working with geometry. I mean, in practice we don't think if the angle is 90 or 44 degrees, we just carve the wood and if it fits well, we leave it as it is. I do it by looking at it through eyes and hands.

When Antonio said "the angle is 90 or 44", I asked him if he had learnt this at Proem. He said that it was a result of learning through Logo, which is taught at school.

In the context of my study, it may appear as if these students had no geometric knowledge. This is not the case, as we shall see when I discuss the tasks these students were actively engaged in. But what is at issue here is that when these students are asked to speak about geometry as abstract school knowledge - as opposed to their own practices with geometry - they do not have much to say. Almost all made no connections with their everyday practices. Antonio did but only to contrast school and work. This might reflect the limitations of interviewing, rather than the capacity or geometric imagination of the students. I shall return to this question in the conclusion of this chapter.

Let us now consider what the manual workers had to say when asked direct questions about geometry. Observing their responses, we may say that most of them, like the PROEM's students, had little comment to make. But unlike the PROEM's students, most of the manual workers were able to say that geometry was 'something' related to mathematics. For example, the carpenter Sebastian stated that:

I don't know exactly what it is. But seems to me that it's a part of maths. I studied very little about it, but as far I remember, it deals with measurement in which you need to memorise the operations. To be honest, it is a mystery to me.

Sebastian then considered that mathematics was an intellectual exercise with little concern for the real world. In his view, geometry was valuable for those who took pleasure from thinking, that is, were interested or able to deal with things of the mind rather than practical matters. In comparing both kinds of knowledge:

In my case, I think I never utilised the calculations taught at school because I can't even remember what I learned about those figures over there. Of course we do use calculations but it is different from those taught at school. The calculations made by a carpenter are learned through his practice work. For example, when a carpenter is working with a T-shaped instrument he doesn't know or at least he doesn't think that he is measuring degrees. He doesn't make this relation.

In similar way the carpenter Raymond observed:

I'm not sure what it is but seems to me that when any teacher works with geometrical figures he/she is looking for the calculations, mathematical ones. I don't know what kind of operation they do, maybe they look for the dimension or volume of the figures.

He went on:

As I understand it geometry, as it is treated at school, deals with those figures within the mind which are not used for practical life, on the contrary, the figures stay in one's mind.

Explicit in the voice of Raymond, as with Sebastian, is that according to them, the formal analysis, the intellectual problem solving in school geometry is all that matters, as one of them observed: "at school people learn theories about figures while we, as carpenters, construct the figures". What this statement demonstrates is that theoretical knowledge is, from his point of view, concerned with 'pure' measurement and it has no obvious relationship either to 'doing' or to 'making' even in the core of geometry when there is such a clear 'practical' side'.

Tom, like the other manual workers, had very little experience of formal education, thus his view of mathematics/geometry was closely bound to his practical work and not based on any rational explanation of the subject. Tom stated "in mathematics you have to calculate a lot, you see the forms, there is the rectangle, and the square". I asked him if he would remember what he learned about these figures at school. He replied:

I don't remember, I know that there were many calculations. In my work I don't need to make calculations, the process of making pots is rudimentary in which I calculate using my eyes. I mean, I don't use a compass or any instrument for measuring the piece. For example, when I make a base to a pot, if I perceive that I haven't allowed enough, I join more clay to it until I get the necessary size.

Proceeding from his description about the process of making a pot Tom noted:

If I want to make a pot I have to start building the part on which the rest is supported, that is, the base of the pot. It is a circle that is cut with a knife. The second step is to make the wall of the pots, these walls are rectangular plates or they are clay rolled by hand as with a rolling-pin... these are the forms that I work with ... I mean, although I don't need to make calculations I think I use those forms - rectangles, circles and squares - when I make the pots.

Examining the above comment, we can say that Tom was familiar with a range of geometric shapes - what he distanced himself from was any precision in terms of defining or calculating their properties.

In justifying his uncertainty about geometry, the graphic designer, Luis, stated that when he was at school mathematics demanded a great mental effort, therefore most of the time he used to feel sleepy in his class: "I have the impression that I didn't learn anything that the teacher tried to teach me". He went on:

As I told you in our first meeting, all that I have is elementary schooling and some short courses in design. It seems to me that all I know is through application and experience.

The potter David cannot recall what he had studied in mathematics. As he observed:

I remember that the maths teacher was a very nice person but what she used to say about the content I don't remember. As I see it, mathematics deals with numbers, right? subtraction, addition, and it demanded a lot of mental work and I don't see the point in learning all the theories. I don't know anything else related to it.

In relation to geometry, David stated:

I don't know exactly what it means. I don't know where I heard it but it seems that it is something linked with the earth, right? Maybe, it is the study of the earth's surface, features of a place, the stones, plants, and even the wind.

The graphic designer Mario's relationship with geometry was clearly demonstrated through the following words: "For me, a drawing is a way of playing with three-dimensional objects on paper, this is my affinity with geometry". In relation to his contact with formal geometric knowledge he stated:

Although I studied some geometry while I was working as a design probationer I can't remember what they taught me about geometry. Of course, I use many measurements for calculating area, volume, and for making diagrams but all I do is to use elementary mathematics, but I'm not sure if I know any formula that I learnt.

Analysing the comments made by the majority of the manual workers, it is possible to say that for most of them the knowledge taught at school was an end in itself, it was pursued (or not) for its own sake with no connection with everyday life. It was interpreted as something close to contemplation (theorising).

Observing the responses given by manual workers and PROEM's students we can deduce that the context created in our last interview limited their speech. Asking them to speak about geometry simply recalled school knowledge and terminology. This did not encouraged them to articulate their own views about geometry, which come from a range of experiences. This tension and almost hostility toward the schooling experience was clear in the observations made by PROEM's students, who tended to be fearful of taking risks, inhibited from exploring the unfamiliar or from expressing their own understanding about geometry. In contrast to their previous extended dialogues, they became monosyllabic - they thought that they should give the right answer as if taking a formal test.

As we will observe in other chapters, when a context was created - one of dialogue - which encouraged them to bring in their everyday knowledge, both groups - manual workers and PROEM' students - fully explored their ideas about geometry. A conclusion we can draw, is that all accounts must be interpreted in terms of the context in which they were produced. Indeed, to show this contextualisation of meaning has been one of my main research interests.

6 Analysis of geometrical forms: the model of Brasilia and the wooden blocks

This main purpose of this chapter is to explore different facets of geometrical meaning and how it is not simply a perceptual construction or mental representation as some logico-analytical researchers have claimed. To this end, I shall analyse how formal and non-formal schooling may influence the way people describe and interpret various geometrical forms/shapes.

Since my analysis depends on the concrete textual description of each group interviewed, illustrative examples are presented and analysed. The fragments taken as examples have been arranged into two sections. These sections correspond to two settings explored during the interviews. Their descriptions and interpretations of geometrical forms/shapes will be analysed comparatively in order to demarcate the plurality of messages produced by different viewers. In the face of these variations, my concern is not to identify a well-defined border of equivalencies or oppositions between the groups as if each had an unique and a particular view. Rather, I am interested to locate the diversities, discrepancies and ambiguities *between* the groups and *within* the group, to interpret contradictions as meaningful social indicators constructed within different practices.

6.1 The geometrization of Brasilia: form as an object of social knowledge

The main focus of this section is the model of Brasilia - its space, volumes, surfaces, and meanings. I took Brasilia as a setting in my research design because of its significance to Brazilians, and in particular to my study participants with the geometrical shapes/forms taken from its landscape that were to be used as material for the interviews. My intention was not to investigate the way people understand Brasilia's urban space, but to ask how people understand geometric forms which are contextualized in an outstanding way and how they interpret

them. Before analysing the findings concerning the general questions, I would like to briefly recall how the models' material has been considered in my research and what I am attempting to identify through the model.

The model, theoretically speaking, can be understood as an iconic sign or as visual text. An icon, as we noted in Chapter 2, is a sign representing its object by means of similarity or resemblance. Within this perspective, the signified is frequently discussed 'as if it refers to the reality that signifiers refers to' (thus the model of Brasilia is the signifier while the signified is the area of Brasilia itself) 'rather than to the concept of that reality' (Lewis 1994:22). The assumption that any visual sign is motivated is crucial in this formulation of how iconical signs are produced. To argue this, the image must have a more direct relation with what it represents than either the written or spoke word. Therefore, provided one knows what the central area of Brasilia looks like, "it seems unproblematical, transparently recognizable" (Kress and Leeuwen 1992:107).

My endorsement of Eco and Kress' formulation of the sign as visual text which asserts that it is a 'phenomenon of the external world', challenges the notion that a visual text has a direct and unmediated meaning which is definitive in its entirety. In the following pages I shall demonstrate, through various participants' comments on the model, how the transference of the sign to external reality - which breaks the natural linkage between a thing and the meaning - was constructed by different groups of persons during my empirical work.

Before examining how the interviewees saw (if they did) the implicit geometrical elements contained in the model, I want to analyse how they described the general view of the model. In this, my intention is to examine what the blocks composing the central area of Brasilia symbolised to the three groups.

Functionality within a subjective vision¹

The PROEM's students, conscious of their social marginalization, brought a strong subjective reference to their descriptions of the model. Their responses when asked to evaluate the model (mainly its general view) expressed indignation and resentment. This tendency may be demonstrated by the following words:

I'm recognising these cupolas. I mean this is that place near to bus station, where the President works with his fellows. They talk about problems, but I am not aware about the kind of conversation they have inside that building. We only have contact with them when we work as servants, like my mother, at their houses.

In a similar way, the student Gil stated that the buildings of the Esplanade which he passed once in a while represented the elite.

This plaza should be the place that represents the nation but only rich people has access to those buildings. I often pass through there and every time I go there I keep thinking that in reality all of them are worthless. I don't understand what they do but it seems that they work with laws to be approved and votes, but their votes always favour themselves. What hurts me is the fact that there is not any interest in poor people, they only look at us during election time.

Comparably, all the manual workers frequently mentioned the functionality of the buildings making an explicit comment about the effectiveness of the government. The political signification of the model was clearly taken into account by the group as a whole. This was evident in many of their comments:

Luis, a designer: It is a plaza where is concentrated the governmental institutions ... these institutions should work together favouring the public affairs of the country, however it doesn't work. On the contrary, its seems to me that its engineering has been affected with rust, I mean, it is without any gearing.

The potter Tom: All these departments should govern and rule the country but they don't control even themselves.

For me, these words impart a strong rejection of politicians as representatives of a system unable to respond to the difficulties experienced by the majority of the population who 'really work and suffer'. This point was raised by two carpenters as illustrated below.

¹ Subjectivity is understood through out my work in its social and cultural specificity as opposed to the universal experience.

The carpenter Raymond: This is where the power and the elite are concentrated. However surrounding it there is an area where the large part of the population of Brasilia live, like myself, who really work and suffer.

Sebastian, a carpenter: This plaza is Brazil's symbol where the power is controlled by those who command this country. Those who regulate the rules: taking political decisions, and approving or disapproving the laws. Recently I have perceived how the people - those who positively work - are manipulated by the government who don't know what is to wheel from the sunrise to sunset ... they really don't know what real works mean ... because all they do is to think and talk ... what hurts me is that they have the power of money while us, the workers, do not have enough money for even a good food.

Observing these comments made by the carpenters Raymond and Sebastian, it one notices that their experiences of loss suffered contributed to their view of social differences, the division of workers and non-workers. In conceiving this division, they highlight their position in the productive process as an exploited class who, according to them, "really works and suffer" while the government only "talk, promise, and don't fulfil what they say".

What these comments indicated is that when a person described and interpreted a situation or an object, they assumed an active social position, expressing specific values and, this position, as Bakhtin (1973) would argue, was conditioned by their particular social experience - and clearly influenced the meaning of the iconic sign.

In what follows, I will demonstrate how schooling also is a social practice, influencing responses which differed from those felt by persons without formal schooling.

Functionality within an objective vision

In contrast to the comments made by manual workers and PROEM's students, the secondary school students hardly mentioned the functional aspect of the blocks presented in the model. Furthermore, there was no manifestation of indignation or hostility in their voices. There was, rather, in their observations, almost an indifference as demonstrated in Caio's remark:

The architect who projected Brasilia, I cannot remember his name, had a preoccupation in organising the city with a very regular disposition. For

example, he constructed the Esplanade of the ministries with a definite form that would mean something.

I replied: 'what does it mean to you?'. Carlos responded:

I don't know ... all I know is that the Esplanade is an area where the government's administration is concentrated. These buildings have separate services ... one is the Ministry of Defence while the other is Ministry of Education, and so on.

The students Elisa and Ana exhibited a similar indifference about the function performed by the government administration in the briefest way: "I don't know what the Esplanade means to me, maybe nothing" and "In my mind it is not clear what the Esplanade signifies to me ... or maybe I don't know how to explain it".

Indeed, a certain difficulty to verbalise their point of view characterised the students. For example, in describing the objects contained in the settings, they repeatedly requested: "Can I write it?" ... "The problem is that I want to speak but I don't know how to express the ideas" or "It is hard to explain".

Two examples of this difficulty to express ideas or feelings into words may be clearly demonstrated through the following comments:

Alex: It is interesting that we see these shapes in front of us several times but we don't pay attention to them. Things cross us imperceptibly ... it is difficult ... there is no way for explaining it. We can see, and even touch but it is difficult to speak about what we see. To describe is very difficulty.

Carlos: I always have difficulty in communicating ... sometimes seems easier to deal with extinct people, fossils, plants than real people. Maybe for this reason I choose the career in archaeology or biology.

These two voices, I believe, reflected an effect of the context lived by students whose primary role was the cultivation of the 'rational scientific subject' who should master and control the mind in order to be able to think clearly and accurately.

Indeed, when observing many comments made by secondary students in my work, such rationalisation enjoys a strong presence. In relation to the model, this tendency is well demonstrated by the students' responses to the questions posed. Asking 'how would you describe the general view of the model', I have noticed the students' group disregard any subjective reference whilst describing the model

blocks and the functional aspect was almost absent in their observations. The group of secondary students, as a whole, was rather more inclined to portray the model in its formal aspect as we shall see in the following pages.

Formal aspects

In describing the model, some of secondary students described the blocks in terms of spatial geometric terminology, while others, in describing them, placed more emphasis on the appearance of the object, using plane geometry terminology. Looking at examples of three students as representative of this tendency in the whole group (five), Elisa and Ana's way of approaching the model of Brasilia illustrates the former and Alex's description characterises the latter:

Elisa: They are solids - three-dimensional objects delimited by six faces, each face being a rectangle.

Ana: the majority of these buildings are prisms whose bases are parallelogram and the bases are rectangular. All of them has six congruent rectangles faces, eight vertices and twelve edges.

Alex: these blocks have rectangular bases. The area of the superior base bigger than the inferior one. Also, there is a big difference in their proportion, that is, while the superior base is thin and flat, the inferior is much larger but both have a common feature, that is, the rectangular outline.

On the whole, it is possible to say that while the secondary students were describing the model they failed to contextualise it. In other words, in expressing the blocks as a mathematical construction, the secondary students tended to consider the blocks as abstract spatial structures rather than part of their cultural and physical reality.

However, as the above extracts indicated, the structure seen 'out there' was not understood in terms of the model's properties nor in the mental concepts referred to in the model. Instead, the students' articulations echoed the language of formal schooling. Therefore, I would say that when the secondary students described the model they were not just discussing the structural² components of

² By structural, I mean the composition and arrangement of the pieces that make up the objects.

the model nor their ability to represent objects as 'mental entities' or 'mental constructs' as the logico-analytical approach would suggest. They were, in fact, describing signs which circumscribe the objects, thus transmitting values, practices, and experiences associated to those images and objects by schooling.

In reducing the model to an objective representation, the secondary students tended to emphasise the formal qualities and the geometrical properties of the blocks, contrasting strikingly with the reading made by the PROEM's students as shown below.

Association with everyday objects or familiar situations

The majority of the PROEM's students described the model as if they were viewing the actual three dimensional blocks, mentioning their familiar appearance and the contextual details of the objects. The following examples illustrate how they materialised the blocks as a component of their actual reality.

Maria: I see it in a very real manner as if I were there: high, white, full of windows, green glasses, and it's stands up. They are similar only from outside since what they have inside is different, especially the people who work there are distinct as well as the furniture's.

Gil: I'm seeing a box that is an imitation of a real building which is localised near to the bus station. This one is small while the real one is big. It has four sides and a decoration in front of it, which seems to be doors where the people enter to the building. Those doors are like arches, similar to the church's one near to my school. The decoration is round, a sort of small wheels.

Kelvin: Looking at these buildings we see many windows and doors ... where the politicians enter and go out. Inside of the building are full of paper and people.

Comparing these observations made by PROEM's students with those of secondary students, we can say that for the former group the blocks of the model represent a physical solid with a particular surrounding. It has size, shape, colour, texture, weight; while for the others they represent a geometric solid in which weight, texture, and colour do not feature; only the size and shape are considered.

To say that the PROEM's students described the blocks in a tangible and concrete way, does not mean that their observations were based on the 'perceptual

qualities of the denotata'- that is, the block itself and its physical components - but rather, if a closer scrutiny of their comments will find that their voices again echoed socially learnt concepts.

Significantly, not having access to the cultural codes of formal education, they hardly made any connection with the geometric terminology, unlike the readings made by secondary students. Instead, the PROEM's students concretised the forms that composed the model, giving it a series of attributes drawn from their everyday context. For instance, the flagpole was explained by student Gil as "a piece like a large nail or an ear of corn. Or it could be an iron stick, round, long, and its base is squared"; "this is a piece like a large pin; or it could be a stick: round, long, and its base is squared". In regard to the cupola was noted that "this piece is nice, it seems a broken ball into middle", also "it is like broken onion or a watermelon cut in the middle round and short".

The difference between 'everyday' and 'scientific' concepts involved in the comments made by the PROEM's students and secondary students, concerned Luria (1976) in his study of 'concrete and object-oriented names', that is, 'practical' reasoning on the one hand and 'theoretical' reasoning on the other. However, unlike Luria, the 'concrete and object-oriented names' expressed by PROEM's students are not understood, in my study, as characteristic of a 'low stage of cognitive development', compared to a 'higher level'.

Analysing the Proem's students responses from a socio-semiotical perspective I point to the great emphasis given in PROEM on emotional issues and the development of self-esteem in opposition to the 'cultivation of the intellect' present in other school which motivates their way of conceptualising geometrical objects.

Therefore, like the PROEM's students, the manual workers, also excluded from the discourse of formal education, tended to describe the blocks of the model comparing their features with familiar objects from their daily life. We can see this in the following examples.

The carpenter Rui's view of the Planalto building moved between the physical aspect, and some characteristics typical of familiar objects and situations. He said:

This building is like a wooden box for packing something, it is also similar to a dice, that is, it is a square. In front of the box there is a ornament: these curves remind me some hammock used by a Brazilian northeasterners, who use it for sleeping. I mean, it is like various nets that are open and tie together on a trunk of tree.

Looking at the hemispheres presented in the model the potter Tom observed: "these are like wash-bowls, also like an egg because of its colour". The carpenter Sebastian made similar observations. Looking at the cupola and the flagpole, he clearly based his comments on attributes to familiar objects: "the one turned up seems a shell and the another is like a half egg. The flagpole "is like a concrete pillar with a squared base.

Similarly, the potter David noted the resemblance between the model and a machine whose functionality was a central concern.

This model reminds me of a machine, where there is a group of buildings in which the government's concerns are administrated. I don't know what the people who work there do but as I understand it, each of those buildings have a duty in which the objective is to control the activities of the government. It is like a machine because at each building there are people who are designed to perform a particular task like a mechanical machine.

Observing the above comments about the model of Brasilia I do not find these quotations describe an iconic relationship to the actual model. That is, in my view, there is no equivalence between the signified and the signifier, as the logico-analytical approach describe the iconic relationship. Rather, my material revealed that there was no analogical relationship between the model and the signifieds constructed by different people. Instead, if we closely analyse the different relationship constructed by viewers it is possible to say that the sign produced resulted from their interaction with the social context. This is what, ultimately, framed the three groups' modes of description and interpretation.

Symmetry within static and dynamic points

In describing the model all participants mentioned its symmetrical organisation. The group of secondary students emphasised the presence of organisation and regularity in the model describing its formal features through geometrical terminology such as: plan, axis, perpendicular, angle, and parallelism. This bias is well illustrated through the strategy used by secondary student, Elisa, to assess the model. Her description was clearly articulated in a sequential order where the model was divided into two sets: the foreground and the background. The former was divided into two subsets of blocks - the group of ministries and the palaces - and the latter into the Congress and the flagpole.

When Elisa compared the two sets she tended to focus on relations of similarity between sides, shapes and the quantity of blocks, as well as on relations of symmetry.

In both sides there are six rectangular and identical buildings settled in a parallel row, almost in a perfect symmetry ... the ministries are in opposite direction in relation to the Congress building ... it is like if the buildings had turned ninety degree, and the ministries are perpendicular to the palaces if we consider the biggest straight line as point of reference.

Similarly, Marcelo and Carlos respectively also emphasised the organisation and regularity of the setting:

There is a central axis that divides the plane into two symmetrical parts; in both parts there are six rectangular and identical buildings settled in a parallel row.

It is evident that both are parallelepipeds which bases are parallelogram but their position are different, that is, the ministries are in diametrical direction in relation to the Congress's building. While the Congress is in a perpendicular or vertical position, the ministers are in a horizontal one.

If we compare the secondary students and manual worker's way of describing the model, we may say that the major difference was in the dynamic area which was untouched for the duration of the former group's explanations. Indeed, in describing the symmetrical organisation of the model, all the secondary students portrayed the blocks as if they were looking at it from a fixed point. In contrast,

the manual worker group, as a whole, unequivocally portrayed the model suggesting movement.

I do not understand this movement suggested by manual workers, as an ability to form and manipulate mental images. The intention here is not to compare or measure the participants' spatial ability - as an ability to mentally represent a situation - but mainly to understand the signification underlining their way of describing what they saw.

By considering the involvement of formal and non-formal knowledge, a comparison arises from those two kinds of descriptions which militates against the logico-analytical approach. In schooling, knowledge has a static interpretation in which the outside world is understood as an atomistic composition of situations and objects³. Thus, the world is seen not in 'terms of process, event, and fluxes' but there is an emphasis 'on parts and wholes or distinguishable static entities of an atomistic structure'.

If we interpret the secondary students' utterances, in a Bakhtinian way, as being more than mere words 'in' a mouth, we might say that the static discourse inscribed in school practices affected, to some degree the responses given by them. Therefore, while the secondary students clearly tended to describe the model in a static and distant way, the manual workers' narratives were strongly connected with actual reality, in which a visual representation of the model is conceptualised with three basic criteria: movement, volume and dimension. Three particular examples are worthy to mention. The designer Mario observed:

Consider that your body is upright and then you have to imagine in front of you a straight line. It is a long corridor that almost by impulse will take you to walk. Now, you go until a determinate point of this line. There you imagine two stand bricks, one is the Senate and the other is the Deputies's House, one is on the left whereas the other is on the right.

³ This way of approaching the world is clearly exemplified by discipline of geography which speaking about the multiple and varied components of the land as units segmented and static "continent" is a constant, "state" is a static unity, mountains, praires, and even agricultural land and cities are conceived of as surfaces, lines, and points, that is as static aspects" (Pinxten *et al.* 1983:160).

The movement aspect was expressed by the carpenter Raymond in the following way:

If I had to say to a friend about it I would say: first, suppose you are entering in a church where there is a long passage from which instead of having long seats made of wood, you are going to see rectangular buildings in which these are placed in similar position to those benches of the church I mean, standing in rows.

He went on:

So, you are going to walk through that corridor, seeing those blocks in both sides, six blocks placed in each side. After passing those blocks and as while you go in direction to the altar of the church; instead of seeing the altar you will see two long buildings: one placed on your right side and the other on the left.

The potter David made a different remark about the symmetrical composition of the buildings: "if we put these two rows of buildings on a scale it will be in equilibrium because they have the same distribution of weight".

The PROEM's students also made comments about the organisation of the model, however, in their observations was a clear subjective evaluation not visible in the other two groups. Thus, the remarks of student Antonio on the buildings contained a particular assessment of vertical or horizontal positioning:

I see the buildings of the Esplanade. Those two, placed on the end of the corridor, are the Congress Palace. They are the highest buildings since they have the greatest power and influence over the others. So, the remainder must be smaller because they are submissive and controlled by that one. In my opinion, the things or the people who are in a vertical position have more control and authority over those laid on the ground level.

A similar comment on the buildings, evoking anthropomorphic imagery, was made by Andrea. Comparing the ministries to the Congress Palace, she observed: "they are different because one of them is laid down while the other are upright. The one is like one person who is down to sleep and the other is stands up as if it were ready to go to work".

The student Maria observed the organisation of the model saying: "there are six buildings placed on each side of the avenue; it is like two lines of people queuing - both with the same amount of people - waiting to enter the cinema".

Analysing the comments made Andrea and Maria, it is possible to say that there was an idea of movement, however subtle, since both students correlated the organisation of the model with an everyday, mobile activity.

Differences within the groups

After analysing the responses of the manual workers and the PROEM's students I found two persons differed, in some aspects, from the others in the group. While the majority of the two groups described the blocks presented in the model by their appearance or by associating it with familiar objects, as we observed above, the designer Mario and Antonio, a PROEM's student, responded differently.

In Mario's comments were included geometrical adjectives such as circular, flat, rectangular, which could characterise properties of a block. But he also described the blocks using clear geometrical terminology such as 'parabola', 'six parallel faces', 'parallelepiped', 'ellipse'. This tendency may be illustrated through the following extracts:

This is a white parallelepiped which has approximately 30 cm in its length wise or I could picture it as a piece of lumber that has six rectangular parts equal two by two. Inclusive, its section is rectangular, that is, if it were sectioned it would have a rectangular form.

In both blocks there are arches in front of it, but the arches are not exactly the same. The difference between them is: the Itamaraty's ones is circular whereas the Planalto's arch is based on parabolas and on ellipses.

However, on occasions, Mario shifted from geometry terminology to familiar objects saying, for instance, "it seems like a box of toothpaste ... it is a solid with six parallel faces".

Antonio's differed from other PROEM's students, commenting on the shapes of the buildings with a certain geometrical inflexion, as he emphasised the shape and form of the blocks rather than their texture, ornamental detail or the material, unlike the other students. These qualities were not wholly absent in Antonio's observations however were much less frequent than in those of his PROEM's peers.

Bakhtin's theories help to explain how a subject may shift meaning. That is, within the Bakhtinian perspective, the subject is not unitary, fixed and unchangeable. Rather, subjectivity is socially constructed. Individuals are socially positioned in multiple and contradictory discourses and as a result their subjectivities are multiple, contradictory and plural.

From this point of view, it is possible to say that Antonio's experience as an apprentice of carpentry marked out his distinction from his peers. That is, although they belonged to the same school and had similar socio-economical and educational background, Antonio interacted with another social activity not lived by his peers. Therefore, the shift of meaning demonstrated in Antonio's voice represented his living interaction with another area of activity which involved a different use of language.

In Mario's case, the shifting of meaning exhibited by him may be explained by his trajectory as graphic designer as well as by the type of tasks carried out him at the University where he works. Apart from his other manual jobs, Mario has been worked in different fields and his work at the university placed him in contact with teachers and students' mainly from arts and architecture's departments. This certainly provided a framework for his observations. In other words, the variety of the geometrical languages used by Mario could be interpreted as representing his interaction with different social environments and his communication with teachers and students from whom he has acquired different 'codes' and 'registers' (ie; the language of the student or of the teacher).

Observing the comments made by the three groups in this setting, we may say that meaning is not solely connected with a reality presented through its apparent features. This point of view contrasts with the logico-analytical position. More specifically, when one examines the observer's ability to recognise objects, events, and situations, one tends to focus on the

perceptual/representational aspects by which observers structure events, and from which they derive ideas and therefore ways of describing the world.

My empirical data, however, support a view that elements exist between the object observed and the observer which are ordered and varied representing, as they do, distinct social practices. The comments made by each group do not portray mental representations or simply iconical features of the objects. Their observations indicate different social 'speech types' in which the main tendencies were: the secondary students privileged the static over the dynamic, their descriptions hinged on polarity rather than contrast; to viewing events at a distance rather than close up and to emphasising formal aspects. In contrast, in the manual worker's comments, everyday life objects and situations were major elements in characterisation, contrast rather than harmony, dynamic rather than static. In a similar way, the PROEM's students emphasised contrast and difference; 'subjective reference'; association and comparison with familiar objects and tended to actualise and materialise the elements rather than abstract them.

6.2 The wooden blocks: objects framing meaning

By observing the different situations presented during the empirical work, one may say that the blocks presented in the model of Brasilia were contextualized since they appeared as elements of the architectural structure of Brasilia's central area. Thus the blocks were associated with particular scenario of everyday life. However, the wooden blocks portrayed in the present setting were 'decontextualised' or 'context-free', considering that they, presented simply as a set of blocks, apparently, did not evoke or particularise any element of physical and cultural reality. In saying this, one has to be cautious because this assertion might imply that a viewer will simply have a mental image or cognitive apprehension of mere blocks reduced to physical properties.

Rather than focusing on the relationship between physical representation and perceptual apprehension of an object, I wanted to examine how a set of wooden blocks, as visual signs, might still convey social meaning.

As I suspected, taking formal and non-formal geometrical knowledge as a variable, most of the manual workers and PROEM's students would not use geometrical nomenclature while describing the blocks. The secondary students, meanwhile, anchored their descriptions in the geometrical terminology. In the following examples demonstrate the manual workers and PROEM's students' unfamiliarity with geometrical nomenclature.

The carpenter Sebastian commented:

I don't know the names of these blocks. Although they are geometrical objects, it is difficult to express it in the right terms. It is a matter of not knowing how to place the word, I mean my words could not fit in a definition, there is a lack between my definition of these blocks and the mathematical one. I should look up their names in a book.

After saying this, Sebastian concluded: "in fact, I don't need to know the names of these geometrical blocks ... all I have to know, as a carpenter, is to do it, right?". This clearly suggests that to identify a geometrical block by name or to define it 'scientifically' is not the primary interest for a carpenter. What is important, rather, is to 'transform action on reality' which, in the case of the Sebastian's activity, is to make wooden things. In this process of 'making' or 'doing', as the carpenter Sebastian himself observed, exist related geometrical shapes/forms and problems. However, the way geometrical elements are incorporated in the 'work-related process' is different from those taught in formal school as was pointed out by Raymond, another carpenter: "(...) geometry, as treated at school, deals with those figures within the mind which are not used in practical life, by contrast, the figures stay in one's mind". Another comment about the difference between what is taught and learned in school, on the one hand, and 'practical action in the made-world', on the other, came from Antonio, a PROEM's student and an apprentice of carpentry. In explaining how to make a cylinder at a workshop, he observed that:

This manner of making a cylinder and finding its length only is possible in a workshop. I mean, in a classroom we also used a compass but the manner of dealing with the things is different. Let me explain it: if I use the words diameter and radius it is because I have learned it at school while I work at the computer, but if I say it to my stepfather, who learned the apprenticeship by himself and never attended school, he would tell me 'what the hell is it?'. He may use a compass but the manner of doing it will be different from that learnt at school.

One inference of these observations is that geometrical understanding may not simply be assumed an effect of mental activity. It must rather be analysed in terms of the elements of a particular 'discursive practice' from and by which it has been constituted.

Like the manual workers, the PROEM's students were not familiar with geometric terminology. For example, the student Andrea described a cylindrical block as: "it reminds me of a candle. All I know is that it is a round thing so different from that quadruped". She wanted to say 'quadrangular'. That moment exposed her unfamiliarity with the names used at in school for geometrical shapes. "I studied about it ... I mean the name of these blocks ... but I can't remember their names. All I do remember is the square, the stretched square (rectangle) and that quadruped".

Indeed, the majority of the PROEM's students like the manual workers referred and described the wooden blocks according to attributes of familiar objects which they associated with its appearance. For instance, the cylindrical block provoked the observations: "it reminds me of a bat: it is a round thing, different from that large box"; "this pin has only one round and curved part, I mean it has one rounded side around itself"; "this is a barrel for oil"; "it is like a policeman's truncheon, it is very strong that may it kill a man"; or "it is all round. It is a small axe". Of the hemispherical block they said: "this round block seems a fat person or it may be a pregnant women with the belly in the days of giving birth" or "it reminds me a of bowl we use at home when the rain comes down, there is a lot of leakage in my house".

Actually, the recurrence of 'everyday' concepts occurred many times during the interviews. But my intention here is just to point out some examples

since this 'vocabulary' as a body of knowledge is not the focus of any attention but rather my concern is to examine the ways in which different groups of persons explored the concepts - the knowledge and understanding - and their relationships to the concepts by consulting his/her personal system of beliefs and values.

In attempting to map the main descriptive tendencies when the blocks were presented in this setting, I shall examine how groups approached a general view of the setting in which the following questions were asked: 'what do you see', 'how might you describe it' and 'can you describe any relation between these blocks and those blocks in the model' and if so, what is it'. In doing this, I sought to compare relations of likeness and difference, connections and inter-relations between the blocks. In exploring these relations I was concern with the following aspects: (i) the kind of criteria adopted by different groups; and (ii) possible similarities and differences among their interpretations.

Observing the manual workers' responses to the questions 'what do you see' and 'how might you describe it' I found that the formal aspects of the blocks were not their focal point of attention. Rather, the group, on the whole, made reference to the type of wood, texture, and heaviness as demonstrated in the following fragments.

The carpenter Raymond most appreciated texture and consistency in a wood. According to him "these are the best characteristics of a wood". Actually, the first observation made by him in relation to the pieces presented in this setting was based on this criteria.

These pieces of blocks are made from different wood in which some are soft others are hard, and their textures are diverse. Those rectangles [referring to the cuboids] are from the cedar tree while those pins [cylinders] are softer; and these half balls are lighter than the others.

While the carpenter Sebastian was looking at the blocks, the type of wood was again the first aspect mentioned by him. He said: "I see several wooden blocks". Holding a spherical block in his hand, Sebastian observed: "I don't know what kind of wood this is , it seems like pine. It is a interesting wood because it is very

light" [he was referring to the sphere which an English's carpenter made using a wood that is not familiar to him].

These descriptions demonstrate that the visual sign expresses meaning not only by their appearance but by what the persons know about it. This understanding clearly contrasts with a logical-analytical approach. The socio-semiotical perspective asserts that the process of conceiving involves a relational connection between an object and a social practice; the logical-analytical approach sustains that the act of conceiving starts with a grasp of the objects' outstanding structural features; an active concern of the mind. What is implied in this perspective is that sign production is motivated by the stimulus of the visual text and a mental process. From the social-semiotical point of view, the motivation between signifier and signified exists *but* is culturally motivated: it is from the perception of the object that one is able to see 'through' it, but from the social code that mediates the act of seeing and the object seen (Eco:1979). Thus, from this perspective, an analysis of a visual representation should not start from the equivalence between image and object but rather from the relation between image and social reality.

For example, the signified (the concept) used by the potter David to describe the signifier (the set of blocks) was not based on the properties present on the blocks. Rather he considered a relationship which reflected an association with the wood produced by labour with the wood encountered in the environment as a natural element. He observed this in the following manner:

I see various pieces of wood: some are rectangular, others are circular. These pieces are not like those found in a natural environment, in a park or in a forest. I mean, these blocks that you are showing to me were made by a carpenter or someone who treated them in order to made their appearance smooth and polished. Those found in the parks are not cut neither carved by someone.

Considering that the sign is a product of people's interaction with the social environment, let's observe how formal schooling may influence the way secondary students describe and interpret geometrical objects.

Formal aspects

There was little difference, in the secondary students response to the model of Brasilia and this arrangement of wooden blocks. In both settings, their descriptions emphasised geometrical aspects. In fact, as observed by a student, Marcelo: "I clearly see geometrical blocks, it would be complicated to see it in a different manner". This 'clearness' was also observed by the student Elisa who had no doubt about the geometrical qualities of the blocks. As she said: "it is very obvious that they are geometrical figures: different prisms, cylinders and hemispheres". Thus, when I asked 'what can say about it?' all the secondary students elected the formal aspect as their main descriptive criterion. This is illustrated through the following examples in which a cuboidal and cylindrical block were depicted as:

This is a prism in which its base is a parallelogram. The total surface of it is the juncture of four rectangles with two parallelograms. The angles are right and the six faces of it are perpendicular rectangles. That's all I know about that block.

I see a prism whose bases are rectangular. I mean, it has eight rectangular solid angles and twelve equal edges and parallel in four. I may also say that these parallelepipeds have right angles

This block has two plane circular bases in which both circles are parallel, that is, each circle must be in a parallel plane to each other. From any point of the circumference one can take various lines which would be the depth of the cylinder that links the upper base to the lower one. And so, a cylinder is a figure which contains one lateral face - or better a set of all straight lines that fills in the circumference, and so it turns to as solid - together with its circular base.

Analysing these quotations from a socio-semiotical perspective, we might say that the signs - as a category of meaning - produced by secondary students 'reflect' or 'refract' an academic experience where shapes/forms are seen as mental constructs in which colour, material substance and mass are irrelevant. Thus, we may say that what the students 'removed' from the blocks were not qualities inherent in themselves but rather those values corresponding to concepts learned in school mathematics lessons.

It is certainly a fact that a circular base or six faces is connected with a cylinder or a cube. Yet, it does not follow that those qualities are logically or clearly connected with a cylindrical or cuboidal block as was suggested by secondary students. If a geometrical block were by nature so explicit and self-sufficient, it presumably could only always be interpreted in one way. My empirical material raises an opposition to this supposition. In particular, it indicates that a geometrical block - considered here as a visual text instead of a 'stimulus material' - does not depict a single and unambiguous meaning; neither are we free to determine the relationships that we construct and perceive between the objects and our concepts about them. Thus, the meaning of an object is neither fixed nor arbitrary, instead it is the result of our insertion in a certain grouping and certain social activities that provide us with the cultural repertoire of resources with which we work.

Significant, from this perspective, is also the need to observe how formal and non-formal schooling intersect in the voices of the students. As illustrated above, the secondary students tended to reduce the blocks to their formal qualities. In contrast, the PROEM's students tended to discard 'intellectual' considerations; the conceptual world of names and properties was not, as we will see in what follows, their first choice in describing the wooden blocks.

Association with everyday objects or familiar situations

Most of the PROEM's students overlooked the ideal qualities of the blocks, clearly connecting it with a range of familiar situations and objects surrounding them, illustrated with the examples below.

Kelvin: I see various objects of wood: they are little stools - tables, ball, and two wooden washstands. Each of them has different 'looks': two of them are round and long, that other is like a brick, and that one is round like a cap [my emphasis].

Andrea: These blocks are like those for children to play with. Each of them has different contours: two of them are round and thick, that other is low in which the sides are different but the thickness is the same [my emphasis].

Gil: I see various blocks. They are squares like a box, others are round as a ball [my emphasis].

From the socio-semiotical perspective, the above fragments cannot be simply analysed as a perceptual act of seeing in which the 'appearance' of the form sustains the discourse about the blocks. If those above extracts were analysed by logical-analytical perspective, the concepts "round, long, and thick" mentioned by PROEM's students might be understood as perceptual concepts in which the information were drawn from the surface-appearance or the 'figural features' of the objects, rather than based on 'theoretical' arguments involving a logical construction of the object or an analytical abstraction of it. Within this framework, we might use Van'Hiele's ideas to analyse the above comments to be at the Level 1 the geometrical development - since the students have the ability to recognise and distinguish shapes and know some vocabulary. In contrast, the secondary students would be at a higher level of geometrical development (level 2 or 3), considering that they are capable of transcending the surface of the reality through abstract and logical strategies.

In contrast with this perspective, we would say a demarcation of the different strategies used by the two groups is not the internal psychological state of the individuals, that is, their 'perceptual -representational' understanding, but their institutional context. The moment we begin to examine the institutional context lived by PROEM's students, it is possible to predict their 'preference' for attributes such as colour, texture and physical appearance rather than logical and symbolic qualities. As it was observed in previously (Chapter 4) the PROEM school does not have make it a priority to encourage the students to reflect or theorise nor to prepare the students to work in the 'practical world'. Instead, the main policy is to foster positive attitudes and to "control the impassivity and anxiety" of the students, as mentioned by the teacher interviewed.

In saying that the meanings constructed by PROEM's students represent or 'refract' this particular social experience, I am suggesting that the relation between signifier and signified is 'created at the intersection of the material and

the discursive' (Walkerdine 1988:93). Understanding the meaning of visualisation, then, is a question of recognising social reality rather than forming a mental image or locating meaning within a visual text.

In what follow, I shall observe how the participants related the wooden blocks with those blocks viewed at the model of Brasilia. The issue here is not just the morphological similarities that exist between them that is signification. Rather, my intention was to observe what would come as point of convergence or divergent between the signifiers: the wooden blocks with those blocks viewed at the model of Brasilia.

By examining the manual worker's way of relating the wooden blocks present in the setting with those viewed in the model, I have noted that the signified 'shape/form qualities' were not the referential point of identification between them, but rather the majority of the workers contrasted the type of material along with functionality.

The potter Tom remarked that "they are only alike in the shape of their surfaces but the utilisation that we do with them is very distinct". Similarly the potter David remarked: "we may say that they are alike only in their contours but these wooden blocks are more like those used by children to play with it. The others are real, where the people enter and go out". Yet, "as solids" said Luis, "they are very similar ... the affinity between them are the surfaces in which compose their structure, but their texture, container and mainly their utilisation are all different".

The carpenter Raymond indicated their differences based on their types of material.

The man who projected those buildings based his idea on those rectangular shapes. I mean only their contours since their material differ; one is made of wood whereas the others are made of iron and cement.

Reviewing what type of relation was made by PROEM's students, I have observed that some of them identified their similarity in terms of the shape/form, taking into account not the model but the general buildings of the

Brasilia, in addition they distinct the material, that is, "while one is made by wooden the others are made from cement".

In contrast to the above comments made by the manual workers, the group of secondary students, as on whole, clearly connected the wooden blocks with the blocks on the model by their geometrical features. In noticing their parallelism the majority of the students said that there was a clear correspondence of their geometrical quality though their proportions were distinct.

In attempting to demonstrate some social effects in constructing the sign, I want to recall some observations made by secondary students about the manner in which the discipline of geometry is taught in their school. My intention here is to examine how this specific social activity was incorporated into students' voices while describing the wooden blocks.

Although the themes and the topics proposed by the Secretary for Education for geometry are not confined to geometry but have relevance across the mathematics curriculum, I observed through the interviews that in practice they are taught separately and independent from one another. Thus, to the majority of students, geometry is composed of many elements such as points, planes, volumes, all divided into unconnected fragments. This atomistic way is explained by students in the following way:

Plane geometry, with its definitions and postulates, is the basis of the knowledge for geometry of the solids. However, the teachers don't make the link between these two geometries; they are treated as if they were two things completely different from each other.

It is not possible to deal with spatial geometry if you don't know the plane. There is no way of separating them because it is very close. However, the students and even the teachers, that is, the approach of teaching the subjects it is like if they were different and distant.

The majority of the algebra is deducted from plane geometry, however, the teachers don't relate this aspect.

If we view meaning of these utterances as a product of social experience, it is possible to suppose that the insistence of the secondary students on separating the wooden blocks, as "if they were completely different and distant from each other", rather than connecting and interrelating them, is an effect of the way in

which geometry is taught in school. In accepting this possibility, we are rejecting the assumption that the way of categorising and classifying objects is only the effect of a logical and mental interaction between the individual and the objects.

In what follows, I demonstrate how the atomistic discourse transmitted in secondary school is echoed in the student's voices.

Polarity

The secondary students' approach to interpreting the blocks was directed almost exclusively towards shapes/forms in where these were grouped together by their differences; expressed in binary oppositions such as: 'this is a cuboid', 'that is a hemisphere', 'that is a prism', 'this is spherical', 'this is totally different from that'.

This last observation, 'this is totally different from that' represented indeed the most remarkable tendency of the students in describing the wooden blocks, showing a clear tendency to polarise objects. This is illustrated by Carlos's observation when he said: "looking at these blocks I see that each of them has a particular characteristic". At that moment I asked him if it would be possible to relate them. He replied by saying:

They are totally different because each one has its own characteristics, I mean, that cylinder it is a cylinder with its properties as well those cuboids have their own features. If I had to compare them I would say that cuboid has something similar to those cubes, that is, they are straight with right angles, that's all I can see. However, that cylinder is totally different from a cuboid, one is curved while the another is straight. Thus, I can't see similarities between them. The hemisphere may be compared to the cylinder even though they are distinct. In my opinion each of these blocks has its own style.

Elisa, in a similar way, polarised the elements present in the setting in which they were grouped into four subsets. Each subset was handled as a unit with no intersection between the sets of blocks. She said:

If I had to sort out these blocks I would select them according to a single criterion: by their shape. Thus we would have four sets: 1. four cylinders; 2. three cuboids; 3. four cubes; 4. one sphere and two hemispheres. We may say that each set of blocks represent an exact type of geometrical appearance and properties. Thus I would place all of them together since they are divergent.

Comparing the cylinder with the cuboid, Ana also followed the criteria of polarisation:

This cylinder is entirely different from that cube: it does not have eight rectangular solid angles neither are its faces parallelogram as these are; thus I may say that the cylinder is opposite of the cube as well as this cuboid.

Alex pointed out the similarity between the cube and cuboid saying "they are of the same type, that is, they are polyhedral. In fact, the cube is a regular polyhedron like a octahedron, while the cuboids are irregular ones. Thus, we may say that the cube is a special case of a cuboid". However, he stated:

We cannot say that the cylinder and the hemisphere are similar to a cube neither to a cuboid. By contrary, they are entirely different since their features, as solids, are very distinct, I mean, number of vertices, edges, faces, symmetry.

When the students related one or more blocks, it was based on the similarities of certain properties, thus, they made connections between cubes and cuboids along with comparisons between hemispheres and sphere. Less frequent, but present, was the mention of circularity as a common feature of the cylinder and the hemisphere. What seems clear is that the secondary students compared the blocks in terms of logical relations according to criteria of classification.

Contrast and correspondence within a constructive description

In contrast to the polarisation shown in the secondary student's responses, the manual workers demonstrated an evident tendency to contrast and correspond the blocks. This tendency was not based on logical qualities rather the relationships were established among the blocks a the transformation of a physical action in which the blocks change from one structure to another.

The possibility of changing one form/shape into another was explained by the potter David in the following words: "if that ball [sphere] were made of metal it would be possible to smelt it, obtaining a cylinder I mean it is workable to change that ball into a cylinder". He scrawled on a paper the below drawing.

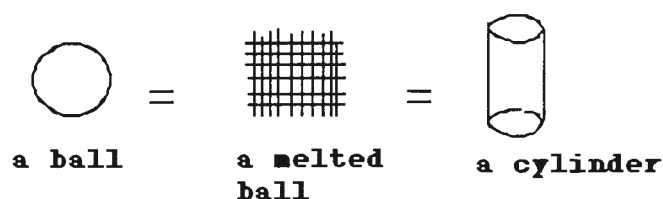


Figure 6.1 Changing one form/shape into another

After finishing this description David made a kind of 'volumetric analysis', involving the measurement of a cube and a rectangle and suggesting that the amount of mass contained in a cube is bigger than in a hemisphere.

If we build a cube using a clay and then I want to stretch it into a flat and rectangular form the mass of the cube is still the same. But if I have a half ball of clay and I want to make from it a cube I would need more material, I mean, the amount of clay for making a cube would be greater thus we may say that the cube would have more weight than the half ball.

What is observable in these two comments made by potter David is the tangible way of describing a geometrical block, in a contrast to the generalisations and abstractions formulated by the secondary students. Indeed, as we can observe in the following examples, when the manual workers responded to the geometrical objects present in the setting, they tended to link the blocks as if they were producing objects in much the same way as they did in everyday life.

At first the carpenter Raymond said that a cylindrical block was different from a cuboidal one, but:

if we have a spinning wheel it is possible to make a roll [cylinder] like this using that rectangular [cuboid] block; even by hand I can do it. I mean, it is possible to carve that rectangle [cuboid] by cutting away the wood until a roller is formed. For it I take an compass locating the centre from the top side of that rectangular solid, then I find the maximum peak of the surface, always touching the edge of it. Usually I do it making using a ruler or a stick. After I fix the compass on the centre and move round the compass until the circle be completed. Lastly I cut the wood following that trace.

Similarly, the carpenter Sebastian observed that it was possible to make a cylinder from a cuboidal block. His method of moulding a cylinder was quite similar to the carpenter Raymond:

If we want to make a pin [cylinder] it might start from a rectangular solid block. Let's see ... the first thing to do is to measure (say a cm measure) the

size of the block, measuring the distance across it. The middle of that distance is going to be the centre of the piece. The second step is to find out the circle of it. If we don't have a compass it could be substituted by three pieces: a string, a pencil, and a nail. All we have to do is to fix the string to that nail, and hammer the nail into the centre of the block. Then, we make a cross and with the pencil that should be fixed on the string we can make a line on the surface of the block (turning the pencil) in which it would mark out the circle. So, knowing it we can carve the block and do the pin.

This way of describing an object - as if they were moulding, handling or shaping the object described - was the most remarkable characteristic of the manual worker's comments. If we interpret this manner of describing as a 'form of cultural production' it is understandable that the manual workers - as sign producers - would conceive of geometrical material in a tangible way rather than in any symbolic form.

What is at issue here is the construction of two signifiers: geometrical object and a practice rather than a relation between an geometrical block, as an iconic sign, and a meaning derived simply from itself. The interviews here demonstrate how the signification constructed by manual workers is deeply connected to their specific form of labour.

For example, if we look at the description made by the potter Tom about a spherical block it is possible to observe that instead of naming and classifying by 'intellect' he moulds the sphere using a familiar material, demonstrating his 'preference' for materialising the geometric form.

Grab an amount of clay and thread in it various pens (around thirty or fifty) of the same size. The pens should be placed one by one in opposite way. At the end, it there would have a spherical surface where the rays will be the same distance [having an equal radius] but in different positions.

This 'palpable' manner of explaining geometrical blocks was also the path traced by the potter David. He described a cylindrical block constructively, that is, as if he were moulding or shaping the block described.

Well, pick various straight wires with the same length and two ring wires. Fix the wires at the edge of each ring in a vertical or it may be inclined position, it doesn't make difference. The wires should move round the ring until closing the curves; the complete curve makes the body of the tube.

The figure below illustrates his description.

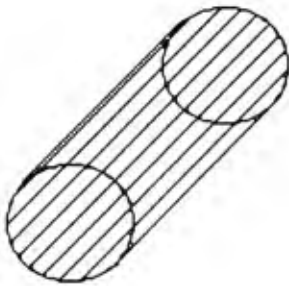


Figure 6.2 Moulding a cylinder

When I asked to the carpenter Sebastian how he would describe a hemispherical block, he replied saying:

I know how to do this block but I'm not sure if I can explain it to you ... it is difficulty to put in words this object because all the time I place it in my hands without using theories.

Sebastian concluded saying that the simplest way to explain how to make a hemisphere is:

to find the central point, that's all you need. It is not necessary to know its height since I know the across distance and the centre of the piece. So, the height will have the same measure of the across distance of the circle. If we have a piece like an egg the distance from the top to the bottom would be larger than the across distance.

After saying this, he made a comment suggesting a distinction between two worlds: that of words and that of action which was said: "from my point of view this explanation doesn't mean anything if the person who I explain doesn't have the skill for doing it" [my emphasis]. In my view, what underlie this commentary is his distinction between the theory - the words - and the practice - the construction. One is based on the concrete and tangible, making by hand, whereas other is made 'by head'. In his opinion, the words do not have meaning if the person does not know how to act, that is, the words fail unless it moves beyond the theory.

At first the graphic designer Mario related the cylindrical block with a familiar object saying that "this is a tube; for me a tube and cylinder have the same meaning. I also may identify it with a more familiar object saying that it is

a can of beer". Nevertheless, in resorting to the 'constructive description', Mario explained:

It is a rotation of the rectangle. Let's say it in different way: imagine a round and large pan full of butter with a degree of thickness or solidity. If a blade - like a piece of paper made from metal - is rotated... here we have to stand the blade vertically and controlling it firmly upright as if it was fixed in a axis or in one of its edges. Then, the blade is rotated and will outline, in space, a circle. In our case, as we are using a rectangular blade, the complete trajectory of it will map out a cylinder.

He sketched the following picture.

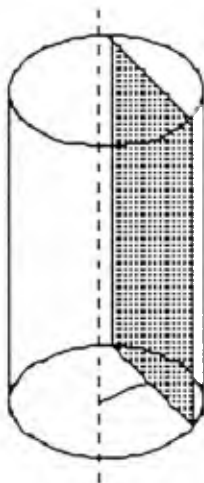


Figure 6.3 A constructive description of a cylinder

Continuing with the same procedure, Mario explained other figures saying that:

the same occurs with the hemisphere because it is also a solid of (...) I can't find a word for it (...) I mean, they are orbiting solids. Let's see another example, a helices of a fan which its trajectory is a circle, or if you take a triangle and turns it, in space, it will become a cone, that is, the trajectory of it, in space, is a cone. Other example might be a square, that is, if it is twist, in space, it may become a flatten cylinder. This is my manner of understanding the solids.

Significantly expressive is to compare this 'practical' way of explaining a geometrical solid with the formal manner described by two secondary students.

The height of this cylinder is the distance between those two planes - the two bases - and I could say about its area ...let see if I still remember it ... it is the sum of the lateral area with the area of the bases. Also, there is a volume, it is the area of its base multiplied by the perpendicular distance between the planes.

This [a cylindrical block] contains two circular bases. These bases are parallels and for this reason they are equal. The height of this cylinder is the distance between those two planes - the two bases - and I could say about its area ...let see if I still remember it ... it is the sum of the lateral area with the area of the bases.

Contrasting this way of portraying a geometrical form with those depicted by manual workers, it is possible to remark that while the students described the form as if it were an abstract spatial structure whose characteristics were taught to them and which they had to remember, the manual workers transformed the ideal quality in a palpable matter, that is, in a form which occupied a physical space rather than a symbolic space. The motion aspect recurred, as the above fragments reveal, contrasting thus with the motionless secondary students' account.

When comparing the manual workers and the PROEM students' way of conceiving, we can see that the second group did not mould or shape the objects like the first, except for the student Antonio, who also was an apprentice carpenter. However, as in the manual workers' comments, movement was a recurrent focus in the PROEM's descriptions and they had a clear tendency to contrast and compare the blocks instead of polarising them in the manner of the secondary students.

In contrasting the blocks the PROEM's students directed attention towards appearance, associating the wooden blocks with everyday objects "this is like a dice", "it seems a box of matches", "this round block seems a fat person or it may be a pregnant women with the belly in the days of giving birth". Similarly they said: "it seems as if we were looking directly at a full moon". Yet, by contrasting the cylinder with a cuboid the student Gil observed: "their parts, I mean, their divisions are different and another difference between them is that a pin rolls while a box does not".

What these examples demonstrate is that even when people grounded their interpretations on the appearance of the objects, they did not base their observations on physical properties but rather produced a *perceptum* based on their interactions with the real world. In other words, as the PROEM's students did not possess formal knowledge about geometrical elements. Their

conceptualisations arose from their relationships with spatial objects encountered in everyday life.

A Dynamic point of view

The PROEM's student's observation "that a pin rolls while the box does not" leads us to examine another point of divergence among the three groups. I have noted that mobility was sometimes mentioned by the PROEM's students while they were describing the blocks; whereas it was almost always absent from any of the secondary students' descriptions. In fact, the student Marcelo was unique in mentioning motion while describing a hemispherical block. In contrast, the aspect of movement was frequently present in manual worker's descriptions. There were three examples in which it was evident. The first one was mentioned by the designer Luis when he compared the cylinder to the cuboid saying that:

One is like a cotton reel that if you throw it to the ground it is going to roll down, over and over, until it is stopped by a stone or other obstacle encountered on the road. However, that rectangle [cube] is different because it is like a dice. If you throw up a dice it falls but it doesn't roll away on the street, that is, it can roll more in the air than on the level ground. Thus a pin rolls while the rectangle doesn't move so easily.

The second example featuring movement was David's observation about the difference between three elements: a sphere, hemisphere, and a cuboid.

The first is like a ball because if one touches lightly upon a part of it, it is going to move away by rolling. However, if one touches that half ball it would move only from one side to side like a rocking chair, and that box wouldn't move as readily because we have to move it by pushing.

The third example in which movement is apparent was mentioned by the carpenter Raymond who said:

Suppose we throw one of those blocks on a lake; we will see on the water various circles in which it enlarges as while it moved on from the point where the block dropped down.

Here, again, it is possible to see that the geometrical form acquires a physical and mobile reality rather than a symbolic and static one. These two tendencies, as I say, are connected with social practices - "cultural clusters which guide and limit the individual's interpretation of messages" (Morley 1994:118). Images,

ideas, and ways of conceiving, then, become attached to practices rather than to perceptual constructions or mental representations of the objects.

Similarities across groups

My empirical material has revealed that there were points of conflict and divergence between and within groups. Nevertheless, as we will see in what follows, there were certain similarities across groups. The point of convergence between the groups occurred mainly when they described the spherical/hemispherical and cylindrical blocks.

The most evident similarity across the groups was their finding the cylindrical and spherical/hemispherical blocks the most difficult to analyse. The reason for this difficulty however varied across groups as demonstrated below.

The secondary students had a school-based reason. In explaining this difficulty, secondary students commented that spheres/hemispheres in solid geometry was the last topic to be studied in the mathematics curriculum. Due to time constraints most of solid geometry was frequently missed, so most of this group had little instruction in this area, rendering all groups equally unfamiliar with the logical relations defining these objects.

The PROEM group of adolescents, with almost no formal study in geometry, justified their difficulty in analysing the cylindrical and spherical/hemispherical blocks by declaring an unclear delimitation of edges, which they found confusing. The manual workers, particularly the carpenters and designers, had reasons from their practice. They explained this difficulty by their familiarity with straight objects and their unfamiliarity with round ones. What underlie these explanations is that particular experiences produced specific readings.

In what follows, we can observe these difficulties explained through their own words. For instance, the carpenters Raymond and Sebastian observed the cylindrical and hemispherical blocks compared with the cubes saying that:

That dice [cube] is easier to explain ... I mean, I don't know what to say about those circular shapes. The explanation for this difficulty may be because at the carpentry, where I really learn to treat the shapes, I make, most of the time, only straight objects.

I can't say more things about that pin and that half ball... maybe because in my work I only produce straight pieces.

The above observation made carpenters Raymond and Sebastian- that the 'circular' blocks are more difficult to explain than the cuboidal one - was mentioned by three PROEM's students. The obstacle quoted was the unclear delimitation of the edges as these fragments below demonstrate:

I would say that their form is less definite than those squares [cubes] because these are well delimited. I mean, the sides of it are equal and the divisions between them are well defined. So, if you look at it you see where the divisions of its parts are, there is no illusion while that pin and that half ball demands more reflection in order to find out how many parts it has because it doesn't have clear corners. For example, looking at that pin, I would say it has three parts: a round one and two tops which are two rings.

These circular blocks are difficult to explain. I mean, we may say that their top is like a wheel but I don't know how to explain the whole body of the block. Or better, I don't know how many sides it has, I wonder if it has sides as those rectangular, because all I can see is an all round side. It isn't possible to see the complete round of it, that is, from this position I can only see the frontal side of it and if I were behind it I would see the another side but now it is out of sight.

Like the other the manual workers and PROEM's students, the secondary students expressed their difficulty in describing cylindrical and spherical/hemispherical blocks.

Another similarity encountered across the groups was that while the participants tried to describe the hemispherical block, all of them compared the dimension of the hemisphere's circumference with that present in the cylinder. Although the comparative method used by different people was similar, differences can be identified in their discussions as shown below. A secondary student made a relation in the following way:

Their dimensions are different, I mean, the circumferences present on the hemisphere change their dimensions while it moves from up to down. In this movement the radius reduce. In contrast, the circumferences present on the cylinder are constant this is, they have equal dimension. Because of it they have the same radius.

Another secondary student described the hemispherical block as overlapping circles which progressively diminished in radius, until arriving on one point (see figure below). In saying this, the student compared a property of the hemisphere with that of the cylinder, observing that there will be a moment in which one of those circles which compose the hemisphere will have the same radius and diameter as the cylinder.

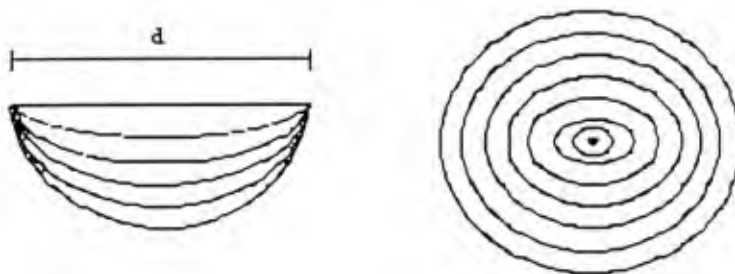


Figure 6.4 Comparing the dimension of the a hemisphere with a cylinder

A PROEM's student, like the secondary students, brought explicitly into focus a link between the cylinder and the hemisphere.

These two blocks are round. One is long and the other is short. That pin [cylinder] has two rings: one on the top and another on the bottom. Both of them have the same size. However, looking at that upper ring of that half ball it is possible to say that it is larger than the bottom one. Therefore, the above circle get smaller and smaller while it approaching to the ground.

In similar way, another PROEM's student contrasted the cylinder with the hemisphere saying: "that tube has various rings. They have the same measurements but the rings of that half egg decrease as they moving closer to the ground".

Analysing the way the secondary students and the PROEM's students contrasted a hemisphere with a cylinder, it was clear that their methods of comparison were not directed towards the external forms of the blocks as if there were iconic signs. Rather, we notice that the codes of recognition came from their social milieu. For instance, when the secondary students described parts of a cylinder and a hemisphere they used expressions such as 'radius', 'diameter',

'circumference', 'segment'. These certainly did not flow from the physical structural features of the blocks but rather from concepts brought from their social experience. That is, it reflected and embodied a set of values derived from students' experience with formal academic geometry. In similar way, concepts such as 'round', 'ring' 'long' 'tube' expressed by PROEM's students reflected their experiences with geometrical elements as elaborated and constructed in their everyday life.

If we accept that the signified does not arise from the physical structure of the objects neither is it a mere mental concept, but rather it is conditioned by social activity of the participants involved, it is possible to understand the manner that the manual workers described a hemispherical and cylindrical block.

More specifically, like the students, the manual workers compared the dimension of the circumferences contained in the hemisphere with those present in the cylinder. However, while the students tended to visualise that alterations by verticality - from the top-down movement - the manual worker observed that alteration by a spiral movement as the comment below illustrates

The movement of that block [hemisphere] commences by the round of its greatest circle and ends in the middle of that block [hemisphere], after making a spiral movement. That movement is clear when I do a pot.

In my view, this spatial movement visualised by a worker demonstrated how a social activity generates and provokes a particular sign. This social motivation in the production of sign became explicit when the participants related the wooden blocks with their environment and everyday life.

Relating geometrical shapes/forms with its formal aspects

Analysing the social/cultural context lived by the secondary students at school it is clear that the way geometry is practised has isolated its themes and topics from the circumstances and the events of the real world. Rather, the physical properties of the objects are unimportant to geometrical discourse since it is not

concerned with applicability to practical problems but with the idealisations the objects.

Indeed, the geometry taught at 'Westside School' is explained and presented as a deductive enterprise with an emphasis on logical and formal aspects drawn from an abstract axiom system in which spatial objects encountered are irrelevant.

In my view, this way of operating and organising mathematics/geometry within the classroom - in which there is no actual engagement between the real world and the academic - may explain the difficulties encountered by secondary students in relating their everyday life situations with the geometrical material displayed in the setting. What this assumption suggests is that the ways in which students described geometrical elements relates to broader factors such as, in this case, formal schooling. That is, the school, as an important centre of formation and dissemination of social evaluation, distributes and legitimate forms of knowledge, values, languages, and frames for understanding which guide and constrain the student's responses.

Thus, the complete dissociation of the academic world from real life processes and concerns, may elucidate the difficulty demonstrated by Carlos, a secondary student, during the interviews when I asked if he would recognise in his environment the geometrical figures of the wooden blocks that I had showed him. Carlos associated them with some familiar objects mentioning that "a TV or a bookcase would correspond in shape to a parallelepiped and a hemisphere is similar to a bowl".

Nevertheless, he had difficulty in making any connection with school work. Carlos had spontaneously mentioned that he has been studying the geometry of solids:

I have seen solid figures that have three co-ordinates, that is, three axes: x,y,z. Usually, the teacher draws these figures on the blackboard and sometimes he/she shows some plastic and transparent geometrical blocks to demonstrate.

Thus when Carlos mentioned the TV set as being a solid object I asked about a possible link between the form of the TV with the three co-ordinates he had mentioned before. Carlos replied, "I don't see any relation between them. Well, it is difficult to see those three elements - x, y, z - it is very abstract. I inquired if he could relate these axes to a plane. Carlos's reply was: "yes, this I know, it is easier. For example, if one draws a square these axes are the width and the height of the figure. In this case it is not necessary to define the length because it is a plane figure". After saying that I went back to the example of the TV set asserting that 'it seems to me that there is a relation between that item and those three axis, I mean, in the same sense of the drawing that had you mentioned. What do you think?' I asked. Carlos remarked:

I don't think so ... we are talking about two different things, one is the TV that I have at home other is about spatial geometry; there is not a relation between them. Well, right now I'm confused, I mean, the spatial geometry studies the forms, right?. But I don't know how to link them.

In my view Carlos did not perceive any connection between the three co-ordinates - length (x), height(y), and width (z) - and a concrete object; suggesting thus the connections were only made within a mathematical world and a real one.

A similar difficulty was encountered by Ana, another student, who was asked if she could identify the figures in her environment. Ana's comment was: "Let me see ... it is difficult. I am sure that some place must have all of them but I cannot remember". She took a moment to think and said: "In the park". Then I asked if she went frequently there? "No, I never go there but I know there is a children's toy there but I don't remember the name of it. However it has the same shape of that round object" (referring to a hemispherical block present in the setting).

The student Ana also commented: "I had never realised that relation of Brasilia [the architectural features of the city] with those shapes/forms taken at Maths classes". One possible reason for this lack of connection, is that school mathematics, as Ana observed, is so concentrated on a book-bound activity with no connection made with the world of concrete objects in all their variety of

shapes and sizes. Thus, it is possible to spend a lot of time calculating areas and volumes of cubes, cylinders, prisms and so on without ever mentioning how these forms/shapes and its concepts might be contextualised for example in the architectural feature of Brasilia.

A similar observation was made by student Elisa who commented upon the textbooks used in schools mentioning that most of the exercises and examples supplied had no practical application. There were "geometry, demonstrations and proofs" in which Elisa considered that the "abstract application, that is, geometrical proofs" was easier to demonstrate than a practical one. She illustrated this by saying that "I don't know if I would be able to calculate the volume of the water of the tank in my house". After saying this, Elisa was thoughtful and right after she reconsidered "maybe I could, but I'm not sure about".

This obstacle in applying knowledge to concrete situation is, in my opinion, an effect of the students' insertion in a social practice where the central aspiration is to cultivate the 'broad intellectual', that is, to foster an active mental life such as memorisation and the ability to judge, evaluate, and classify elements of knowledge rather than apply it in any way.

To argue that an individual's interpretation is affected by the social context is by no means to opt for a mode of determinist explanation in which the individual interpretation is directly explained by social position. This would imply a clear-cut opposition or a well-defined border between the groups in the research. Rather, as the empirical data shows, there are points of conflict, divergence and ambiguities as well as similarities across groups. For example, among the secondary students there was one, Marcelo, who does not have any difficulty in connecting geometry to Brasilia. In Marcelo's opinion the architectural structure of Brasilia is helpful to understand the geometry of solids, saying:

Brasilia is a perfect city for those who want to learn geometry - especially 3-D - since everything is geometrical. For example, the four points of the compass

- North, South, East, West - are the main reference within of Brasilia where everything is based upon this. I mean, all in Brasilia (streets, places, building flats, houses) is grouped and named by those cardinal points. It has also a strong influence because we are in a frequent contact not only with geometrical surfaces but with the way these surfaces are arranged. Thus, as I understand it, when a teacher is explaining about the geometrical shapes, it is possible to associate and imagine those figures with the things of Brasilia. The variety of geometrical objects in Brasilia is enormous.

When I asked to Marcelo to make a free drawing by using the material present in our interview, he drew a map of where he lived. The intention was to demonstrate the geometrization of the place (see below).

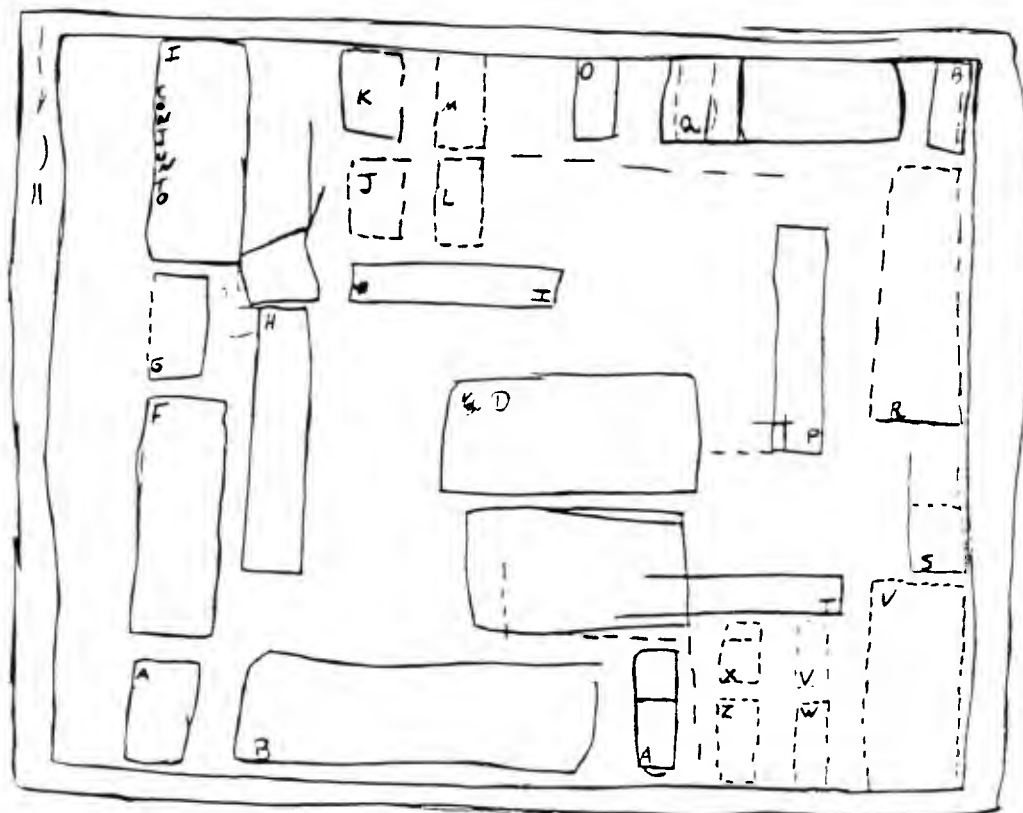


Figure 6.5 Geometrization of Brasilia

Marcelo explained his drawing saying that:

Everything in Brasilia has a symmetrical order. Although I don't live in the central area of Brasilia, the local I live it is very similar to it. The houses are grouped in an area that are called 'quadras' that mean squares. Each set is composed of eight houses, positioned in an ordinal series: first, second, third Also, the streets are parallels arranged, and the sets of the streets are rectangles as well as the houses themselves. The houses follow an alphabetical order: A B C D ... nothing is random. Therefore, there isn't any doubt that Brasilia was created based on a geometrical order.

As we will observe in the following pages, this strong connection of geometry with a concrete reality noticed by Marcelo, was unusual for his group who had a strong tendency to describe the settings only using a geometrical description.

Relating geometrical shapes/forms with everyday life

Unlike most secondary students who saw geometrical objects separated and detached from living contexts, the group of the PROEM clearly established relations between geometrical forms and their everyday life situations.

If we review what is the main policy of the PROEM we recall that the transmission of formal knowledge is not their main purpose. Rather the patterns of socialisation, aiming at "self-esteem, positive creativity, personal behaviour towards school and community" are emphasised. Given this social context lived by PROEM's students it is hardly surprising that they make few connections with geometry terminology while describing the elements presented in the settings. Indeed, the group tended, on the whole, to describe the components of the settings by their appearance, by association with familiar objects, and by subjective references. Therefore, reference to their home environment is intrinsically linked to their descriptions throughout the interviews.

When I directly enquired if they recognised the wooden blocks in their home environment all the students had no difficulty in making any connections. As illustrated below, this group of students tended to associate the geometric forms with familiar objects and everyday situations, as the following observation made by student Antonio who related the wooden blocks with the city of Brasilia. He:

In Brasilia these blocks or better their type of shapes are encountered in many places ... in the buildings, parks and through the avenues. In the case of the buildings they are more alike those wooden blocks since they have mass but in the avenues the shapes are flatten. I mean, the streets have a long square form but they are dismantled because they don't have walls as the buildings or like those wooden blocks

In a similar way, Kelvin observed:

I see these kind of contours in many places. At school there are a lot of pieces which are squared ... such as a shelf, a desk or even the classroom which has

four walls, the difference is that we can enter in the room through the door or jump a window or from inside we can see the panorama, the trees, the basket court out there, in the case of these blocks we can not enter inside it, there is not hole for doing it.

Observing the above commentaries made by PROEM's students we may note that the geometrical forms are not defined as static units but rather the emphasis given is on the dynamic aspects of the elements. This kind of interpretation clearly contrasts with the secondary students. Yet, another contrast observed is that, while the secondary students tended to emphasise the two-dimensionality of the forms/shapes, the PROEM's students inclined to actualise the forms/shapes volumetrically.

If we read this differentiation in Bakhtinian terms, it implies a recognition that the way we perceive the events and happenings surrounding us is a result of our insertion in a certain grouping and certain social activities that provide us with the cultural repertoire of resources with which we work. In consonance with this viewpoint, in what follows, we will observe how non-formal schooling influenced the manual worker's way of connecting the wooden blocks with their everyday practices.

Relating geometrical shapes/forms with familiar situations

By examining the kind of relationship made by manual workers it is clear that the majority of them had a tendency to associate geometrical forms with their work. Yet, as we will observe in the following fragments, the manual workers described the geometrical objects through the senses and the body, rather than holding it at a distance intellectually as occurred with the secondary student's descriptions. This manner of describing the objects is linked to the activities carried out by them where the way of achieving 'knowledge' and 'understanding' involves a clear connection between the subject and the nature which exceeds the limits of the rational and objective. In suggesting that the manual workers are not

involved with intellectual activities as the secondary students, I am not saying that the manual workers are not intellectuals.

The potter Tom who is outside of the educational apparatus, explained the connection between the geometrical forms and his process of producing pots without mentioning any formal aspect. In so doing, he clearly tended to describe the forms/shapes as if he were moulding or shaping the object described. This tendency may be demonstrated by the following commentaries: "seems to me that when I do my pots ... rectangles, circles, and squares are present in my work". In attempting to illustrate this statement he explained the assembly of the pots saying:

The first step is to press and stretch a wet clay with the hands to form a firm smooth paste. This paste may have different forms, I mean, sometimes I do a square other times I make it like a ball or an egg.

In fact, Tom considered that during the process of pressing the clay passes through various forms and shapes, that is, "whenever you press the clay in your hands the shape of it changes because it is in movement, therefore the form and direction may be altered". In explaining this process he observed:

When you are pressing the clay you already know what shape it is going to be but the movement of stretching it the form changes. Sometimes you don't realise it and continue making that shape you had decided, other times you perceive the alteration therefore it may change the initial plan. I mean instead of making a ball you do an egg form.

In a similar way, the potter David connected geometrical forms with his work relating the shapes with the motifs used in the pots and with the pots themselves. He explained this in the following way:

I use those shapes while I build a pot. Let me explain it better, after kneading the clay I do some long screw threads, with them I make various rings in which it is the base of the pots, that is, from this base I start to mould the piece. I could say that those rings are like circles, right?. Then, I make various rectangular and flat pieces with the clay; using these pieces I mould the wall of the pots. I keep moulding with my hands until getting the desired shape. The surface of the pot has to be smooth and uniform. So, those pieces are rectangles like those surfaces present in these wooden blocks, but I also can make a rectangle using circular rolls.

For Raymond, geometry is closely connected with his work however the difference between the way he deals with geometry and how it is taught in school is observed by him:

Everything I do is linked with geometry, that is, with those shapes, but what they do is completely different from the manner I treat the shapes. I mean, those figures are constantly present in a carpenter's workshop because everything we do is based on those shapes, i.e. when I build a closet it has a rectangular form or when I make a table it is based on a rectangle or in a circle if it is a round. These objects will be used by people, for example, a bed for sleeping or a sofa which serves to sit or read on. What I have tried to say is that the value given to those figures at school is unlike because they are not concerned with the use of it.

In a different way, the carpenter Sebastian related those geometrical blocks with some familiar objects. He stated:

I see these shapes all over the place. In my home that rectangle could be the refrigerator or a shelf. Here, in Brasilia, one can find these kind of blocks everywhere, mainly those straight one because the houses and the buildings are based on those shapes. In my home town it would be distinct for the reason that over there we find many variations of the arcades where the style is colonial.

Another correlation was made by graphic designer Luis, who related the shapes with musical instruments used in his home town during parties:

In my village we make a lot of parties where the music is the most important component of it. These shapes reminds me of some instruments. The circles may be the cymbals or a drum, that half ball is like a kettledrum, the cylinder is like a drum stick or a flute. I can not find correspondence to the cube. This is easier to find among objects present at our houses, mainly on the storage.

As these quotations indicate, the manual workers tended to associate the geometrical forms with their immediate experience demonstrating thus how shapes and forms are incorporated into familiar situations or production of the objects. In contrast, as was shown earlier, the group of secondary students had some difficulty in relating geometrical forms/shapes to their everyday life situations; while the PROEM's students easily made connections with their environment.

I have argued, as an alternative to the logico-analytical approach, that these distinct manners of viewing and treating geometrical elements should be related to the practices and modes of life affecting people, rather than a set of particular mental representations or abilities. In particular, my attempt in this

section was to demonstrate how each social activity or practice involving the interviewees represent a specific vision on geometry: formal education geometry, 'everyday-life' geometry, and 'workplace' geometry.

7 Generation of meaning: photographic image and computer screen

The aim of this chapter is to examine how different visual media may motivate certain interpretations and inhibit others. In asserting that the medium may provoke or constrain some interpretations, I am not saying that meaning is an effect of the medium; rather I argue that it is crucial to examine the social and cultural contexts which ultimately guide the way people 'read' the elements in the medium.

7.1 Photograph of wooden blocks: is a photograph 'a mere duplication of reality?'

Suppose I place a wooden cube on a table and ask 'what do you see'? You answer: 'a box'. After this, I show you a photograph of the "same" wooden block. The question here is: 'would you say that the picture is the same box viewed on the table?' or 'is what you see now something different?'

These questions and the reasoning behind them informed the presentation of this setting. In showing a photograph of the wooden blocks my intention was to explore if subjects considered it a mere copy or reproduction of the blocks depicted. In other words, if the photographic images of the wooden blocks corresponded point by point to those shown on the table so that the former represented the appearance of the latter. If so, might we say that the photograph - as a visual sign - had a merely confirmative function in that it simply reproduced the objects?

For Stuart Hall (1972) this interpretation can deceive since "photography **does not** [his emphasis] in fact possess any of the properties of three-dimensional subjects considering that photo is not a mere duplication or reproduction of the reality but rather it is a transformation of the reality" (1972:53-57) [my emphasis].

This process of transformation operates, according to Hall, within codes of social practices that establish how the photo's producer and the viewer should represent and read the elements composing the photo. This concern can be illustrated through an example given by Bourdieu (1965) in *Un Art Moyen*, when he refers to research about the reactions of peasants to a photograph which showed the wrinkled hands of an old woman. The peasants expressed surprise at this result of tiresome daily domestic work, of toil in the fields. According to Bourdieu (Bourdieu 1965:132), what was perceived, understood and valued by them was not the hands of an old woman, but the old age associated with work and honesty.

This assertion allows us to conclude that meaning is not totally determined by the visual itself. However, when I designed the exercise with the photo of the wooden blocks, my expectation was that the way in which three-dimensional elements, depicted in the photo, might produce specific sorts of interpretation related, partially, to the medium. When I asked a friend of mine to take a photo of the wooden blocks I supposed that her intervention, that is, her investment in the sending of a message would construct a different signification for the wooden blocks. A myriad of possibilities of variation lay in the way she would filter, frame, focus, and use lighting to highlight some aspects.

Building on this idea, I shall examine two main points: (i) how the photograph of the wooden blocks was interpreted by different groups; and (ii) how the relationships were perceived and differences noticed between the wooden blocks and the photograph of the wooden blocks settings.

Remarkable distinctions arose when we compare the way participants approached the blocks depicted in the photo with their reactions to the settings observed in the previous chapter - they gave new meanings for the blocks. The first difference was that all the participants focused on the surfaces of the objects and noticed that they looked different according to their position in relation to the camera angle and the use of the lighting in the photography. The second difference

was that the secondary students, instead of polarising the blocks, as had occurred very markedly in the previous settings, related them through contrast. These differences will be analysed in the following pages.

Looking at the cuboid displayed in the right background corner of the photo (cf. page.71), the secondary student, Alex, observed the effect of the light falling upon the block. He said:

Looking at this block we can only see the frontal and the superior base - but the lateral base is not totally shown rather the lateral base comes out slightly and in a progression. By reducing the shade, the lateral starts to appear from the left to the right, till you have a more general view of the solid.

He went on:

If I look at the superior base of the block I can see a cuboid. But if I look at only the lateral base I can see a dihedron, that is, a solid figure having two planes faces. It depends on the angle one looks at ... If I take as basis the frontal face, this block would not be a dihedron anymore but a tetrahedron, that is, a solid figure with four triangular faces.

As this brief extract indicates, the question of the camera angle employed and the use of lighting used by the producer of the photo provoked a new significance for the cuboid. First, instead of strenuously polarising the blocks as before, the tendency in the present setting was to contrast, i.e.; contrasting the effect of the light upon different bases of the block. Second, when the blocks were simply shown on the table the concepts 'dihedron' or 'tetrahedron' did not appear in any observation - they only appeared through the window of the photograph. In my view, this finding supports the two suppositions that I take as a priori: first, a sign - the relation between the signifier and signified - is not 'a fixed semiotic entity' but rather the sign is altered through contexts. Second, the properties of the blocks do not reside only within the image itself but rather are based on one's knowledge of what the blocks are. For instance, concepts such as dihedron or tetrahedron did not correspond to values emanating from the images themselves - from their physical structure - rather they expressed ways of making relationships derived from the school mathematics curriculum.

Taking the involvement in educational discourse as a variable, let us compare the above secondary student's comments about the effect of lighting upon

a cuboidal block with the explanations given by two manual workers. In these, we can observe that instead of focusing on any formal aspects or relationships they emphasised the appearance of the blocks and brought into their talk subjective references as the example below shows:

It is a very nice photo. What I most like in this photo is the distribution of the light. Some surfaces are receiving more light than others so we can see a beautiful contrast among the surfaces of one block or in relation to other blocks.

A different observation was made by the carpenter Raymond:

What is calling my attention in this photo is that when I looked at these blocks placed on the table I didn't pay attention to their grains ... but right now the first thing I saw when I looked at it was their grains ... it is clearer in the photo than before.

Analysing this comment from a socio-semiotical perspective we can say that the signified 'grain', mentioned by him was not motivated solely by the perceptual features of the block. Rather, this characteristic of the block represented his 'particular interest in the object, an interest which is itself a reflection of his place in the world'. Thus, it is conceivable that a carpenter - as sign producer - would be primarily attracted by the textural quality of the blocks rather than by their formal aspects. Indeed, as he himself observed (cf. page. 129), what he most appreciated in wood was its texture and consistency: "these are the best characteristics of a wood".

The following observations again are those of secondary students and manual workers. In them, we can observe how the blocks depicted in the photograph resulted in different meanings from those presented on the table.

The student Marcelo commented on an ambivalence in the way the blocks could be viewed through the photograph: "when we were looking at this hemisphere placed on the table it was clear that it was a 'half ball' while in the photo the same doesn't occur. That is, it is difficult to notice the difference between the hemisphere and the sphere". A similar answer was given by the student Alex: "I see a series of solids, some different from the others ... prisms, cylinders, hemispheres and a complete sphere. From the dark background of the photo these two hemispheres could be a sphere". Analysing these comments, it is

possible to say one signifier can, given the situation, denote various signifieds. Furthermore, this suggestion implies that the photograph was not 'read' by participants as 'a mere duplication of reality'.

In what follows, we can see other comments which suggested that a photographic image was not a mere copy or reproduction. For example, the graphic designer Luis, observed that:

This is a beautiful photo of the those wooden solids. However, now they seem different ... I mean, when I looked at those blocks placed on the table I didn't have any doubt that they were placed on a covering but looking at these blocks on the photo it isn't clear ... the impression is that the blocks appear in a loose group, in a black space.

Like Luis, the potter Tom noted an absence of the horizontal line which would provide a frame of reference in judging the position of the objects. He remarked: "the way these blocks are displayed in the photo gives me an idea that they are suspended and inclined because we can not see the table on which they are placed".

Yet, observing a cuboidal block in the photo [placed on the right top], Ana, a secondary student, declared that:

The top part of that cylinder wasn't provided with light like the others. In fact, the rays of lights come into its lateral base, thus there is a light surface as well as a darker one. However, it is possible to think of it in another way, that is, since that block was made of wood it is feasible that the wood itself had different qualities.

The student Carlos, like Ana, commented:

There is an interesting aspect in this photo. I mean, if this block [hemisphere] was simply floating in space it wouldn't be possible to say if it were a sphere or a hemisphere. However, in this case it is a hemisphere because all objects around it are over the same plane. Likewise that block could be a sphere in which one part could fit on to the table.

Analysing the above comments, I would say that the secondary students' strong predilection to polarise was inhibited by the way the blocks were depicted in the photograph. That is, the focus and the use of lighting provoked a more nuanced than clear-cut view. Moreover, contrasting the way secondary students described the photograph of the blocks with their observations in previous settings, we can see that the present exercise encouraged more subjective references, more

expressions of feelings and appreciation of the aesthetic of the photo as the following observation indicates:

I don't know how to explain it to you but there is a difference between looking at a photo and at the real objects. I mean. it is evident that these blocks are the same as those that were shown to me the other day, but looking at them in the photo they look more beautiful than those viewed on the table.

I think that the contrast of lighting between these two prisms is really beautiful.

The contrast between the hemispherical and the spherical block is nicely demonstrated through this photo.

These subjective references appointed by secondary students, were strongly emphasised by PROEM's students as had occurred in the previous settings. The observation made by student Gil illustrates this tendency:

I see an artistic photo, maybe the photographer is one of those people who appears on the TV in São Paulo, showing their art in the big saloons. Sometimes I see it in the 'Globe' [a TV channel] they work with art, but only rich people have the opportunity to go to this kind of place so I don't know about it. But if I have to say something about it I would say that ... these pieces are beautiful and soft.

Yet, all the PROEM's students' views of the blocks were associated with the lighting reflected on the blocks. For example: "that block [he was referring to the sphere] may be the moon" or "the earth viewed far away ... some pieces are darker than others, for example, that small rectangle [a cuboid positioned on the right] as well as that tube [a cylinder positioned on the right side down] are lighter than the others. Also, that [a cuboid on the left] has a shadow giving an impression as if it were broken".

When the groups considered the possible relation between the wooden blocks presented on the photo and those blocks viewed in the model, it is possible to note similarities and differences. The secondary students tended to emphasise their similarities in terms of forms without mentioning any functional aspects or displaying much subjectivity. Thus they said that the photographed blocks may be perfectly reminiscent of the Esplanade buildings. They observed, however: "we cannot say they are equal. All of the blocks of the Model are present since the architecture done [there] was based on the parallelepiped. But the proportions

are different". The manual workers and PROEM's students also understood the relation between each set of blocks by their forms, however, they added functional aspects as can be seen in the following comments: "they come from the same drawing, but those viewed before have windows, doors, glass, and these are all closed, thus nobody can enter in it". The potter Tom agreed with the PROEM's student saying:

All of them are rectangular this is their unique point in common, that is, they have a similar geometrical style. However, their appearances are different not only in dimension but also in their facade in which there are windows and glasses.

Maria, a PROEM's student said: "the unique likeness is that the man who built those buildings was imitating the appearance of those blocks; it couldn't be the opposite because the wooden blocks existed before the buildings, that is, men imitate nature and it is not nature that imitates men".

When I designed the photograph, as mentioned at the beginning of this chapter, my expectation was that placing three-dimensional elements in different settings would produce different sorts of interpretation. I assumed different depictions would highlight distinct connections between signifier and signified. Looking at the above results, it is possible to draw some conclusions: first, meaning certainly is neither single nor fixed. Rather, it can be changed by the way the objects are depicted in different situations, i.e; the presence of the same geometrical shape in different forms of representation can produce different sorts of interpretation. Second, the material life of the sign does not arise from the physical world as suggested by formalist point of view. Rather, as my results indicate, sign - as a category of meaning - is a cultural expression which represented, in my study, formal and non-formal schooling understanding - the formal emphasising classes of geometric objects, the informal their function, beauty or surface appearance.

7.2 Production of meaning: computer screen

In recent years there has been much research involving geometry and computer. When mathematics education researchers analyse this relation, they tend to enquire whether the strongly visually-based computer environment and its interactive potential facilitate the construction of geometrical representations and understandings. Thus the main concern has been to analyse how action and reflection on the computerised images interact dialectically in constructing geometrical awareness. In considering the relation between 'actor' and screen images, most researchers attempt to understand the type of mental reflections or mental images resulting from the interaction.

In this debate, the computer is often conceptualised as an environment (or 'microworld') which can be used to provide "both virtual worlds of realities and realities materialising theoretical concepts" (Laborde 1995:261). In this way, ideal qualities of geometrical concepts are made more tangible for inspection, manipulation, and discussion. Thereby encouraging "mental experiments of image-reasoning" and a "correct interaction between the figural and the conceptual components of geometrical reasoning" (Mariotti 1994:114-115).

In the present research, as an attempt to offer an alternative view, the computer is recognised as a specific signifying practice, that is, as a means of producing a system of meanings¹. In analogy with Coward's view of film, I see the computer as a "system of signs or images which is (...) not just made up of pre-given relations between those signs before which the individual is passive, but rather a work or process which produces an articulation" (Coward 1976:6). This point of view, which is the starting point of my reflection, is close to those of constructivism perspective. However, my theoretical reflection takes a different route when considering the articulation between individual and computer.

¹ This way of understanding the computer is inspired by, and adapted from the work of Hall (1980,1982) and Coward (1976).

In particular, in contrast to the constructivist approach, my main concern is not to analyse the articulation between 'screen images' and 'mental images' which would imply a connection between perception and cognition, image and abstraction and representation. Rather I focus on an action which is semiotically performed by viewer upon the 'screen images'. Thus, my interest is to analyse how social experiences, priorities and interests intervene in the process of producing meaning. Besides this interest, I shall examine how the computer as a visual medium may motivate different interpretations of geometrical objects.

In the same way as I expected that the photography - as a visual medium - might contribute to the production of particular meanings about the blocks, I also suspected that the nature of the computer's message might provoke certain readings about the forms/shapes. My expectation about the photo's effect on the viewer centred on the way framing, focusing, and the use of lighting affected how the blocks were described. My expectations about the computer revolved around its facilities for depicting geometrical elements in different positions and orientations and its potential for manipulating the elements. More specifically, I had as a priori assumption that the computer would provide a dynamic mode of addressing the forms/shapes not present in other settings, generating thus different modes of interpreting and relating to the shapes². However, as previously suggested, if we want to understand what effect the medium has in terms of the meaning the different groups of persons give to a message transmitted in a particular medium, we have to see how it interacts with their cultural/educational background. Therefore, the focus of my analysis is on the identification of possible patterns which would be associated with the particularity of the medium as well as connected to the specific groups of persons interacting with the computer.

² In this sense, the computer is not a mere system generator or manipulator of images rather there is the "presence of an interlocutor who does not simply receive the message but (...) is asked to complete them, even to change their meaning while they are being deciphered (...)" (Bloomer 1992:55).

On reviewing the material collected during the computer activities, I observed that the way the participants interpreted the elements was clearly not uniform. I therefore divided their commentaries about their strategies according to the activity instead of giving a general view.

In analysing different interpretations produced in the four activities, I first wish to recall four theoretical conceptions derived from the ideas of Bakhtin, Eco and Kress: the polysemic nature of the visual sign; the 'equivocal of the referent'; the transitory correlation between the signifier and signified; and the 'mobility of semantic space'.

7.2.1 The polysemic nature of the visual sign

My interest in this section is to analyse how a set of pictorial figures conveyed meaning; in other words, the concern is to investigate which concepts emerged in relation to the figures appreciated and how they were formulated. Yet, I shall examine possible analogies and differences between a plane figure depicted on the computer screen and a solid object.

In order to analyse the variety of 'readings' constructed by the groups, I shall take two fragments from each group which represent the main tendencies of the groups when describing a set of figures loaded on the screen. These figures were of the model of Brasilia viewed from a plane point of view - the arrangement was not the same as it appears in the real setting (cf. figure 3.3 page 72). The question explored was: 'what do you see'.

Alex, a secondary student: looking at these items in isolation, it is clear this is a series of geometrical figures. But looking at them from this angle it reminds me of a 'distorted' cross without symmetry.

Marcelo, a secondary student: The first impression is that they are geometrical figures, however it is possible to imagine them as a person's face - with eyes, nose, and mouth, made by geometrical figures. It may be an abstract construction.

Kelvin, a PROEM's student: I see the contour of a garden; the flowers are unseen. In the middle, I mean, this round thing is a spring ... where the people can walk around in order to feel more fresh.

Gil, a PROEM's student: I see various squares: small, middle, and big as well some which were stretched. I don't know what they are but that might be the sketch of a game. I mean, those are the squares to play with it ... we can jump from one to another

Luis, a designer: those figures remind me of something about Brasilia: the shapes of the flats, the round-about, and the shopping centre. However, they are distributed in a distinct manner; their similarity would be more clear if these figures were organised in a certain disposition.

David, a potter: I see various vegetable gardens, some of them are squares and others are rectangles. I have one garden like that, where I planted green vegetables, cabbages, and lettuces.

At first sight, looking at the interpretations constructed by different persons we might think that there are six disparate views about the geometrical figures in which each one was simply representative of the observer's own personal view. However, once we begin to look at those fragments the more likely it might be possible to identify particular patterns which demarcate different interpretations one from another and within the groups.

For instance, although all groups recognised and identified the set of geometrical figures as having 'something' familiar, we can observe that the secondary students looked toward a schematic and diagrammatic rendering rather than the 'tangible' portrait suggested by the other groups. In other words, if we review the secondary students' commentaries such as "it reminds me of a "distorted cross" or "it is possible to imagine it as if it were a person's face", it can be said that these expressions suggest a figurative description in which the geometric images were viewed emblematically. For instance, a symbol became a cross or a face, or "it may be an abstract construction". Seeing things in this way, they tended to emphasise formal qualities, contrasting with the other two groups who described the geometric images as if they represented a 'real' solid body. The formal aspects of the objects of their physical world and scenes of their everyday life were wholly absent in their comments.

It is also useful to observe differences encountered within the group of manual workers. For example, the graphic designers, unlike the other manual workers, spontaneously noticed 'something' familiar between the set of geometrical figures and the architectural features of Brasilia. This, in my opinion, is not accidental. Rather, if we believe that a "pictorial space is something we learn to read" (Harrison 1993:203) it is possible to understand the reason that graphic designers evoked that connection more easily than other manual workers, who are not engaged with social activities or practices which involve tasks of graphical representation.

The different interpretations noticed after analysing responses to the first question explored during the computer activity, became clearer when I asked the participants to tell me if they recognised any correspondence between the computerised figures and the blocks viewed in the model. Here, as before, the intention was to examine how the different groups related a plane figure with a solid object.

When observing the kinds of recognition made by secondary students it is noticeable that their arguments again had a strong rational basis; that is, the majority of the students considered both, the screen images as well as the blocks of the model, as configurations occupying a two-dimensional pictorial plane whereas actually, the blocks occupied three dimensional space. In describing the blocks as configurations, the students considered lines and surfaces to be a visual representation of geometrical concepts. In so doing, the pictorial figures as well the blocks gained universal and atemporal qualities in the students' voices. This may be illustrated through the examples below:

As these figures are on a plane some properties change when looking at them in three-dimensions. For example, here there is a square which has four identical right angles, four corners, four edges, one face and no solid. So, this is different from those parallelepipeds viewed the other day on the model ... since those have eight edges, twelve equidistant edges, six rectangular surfaces, or better, are bounded by three pairs of congruent rectangles.

It is very obvious that they are plane figures: different squares, some smaller than others, many rectangles and one circle; they are down on the plane while

those blocks of the model are prisms, that is, solid figures which occupy three dimensional space, therefore, bounded by a closed surface.

This logical and rational way of seeing contrasts to the substantial and tangible sense made explicit in the PROEM' students and manual workers' opinions. Although these two groups, like the secondary students, commented on the dimensionality of the figures and the blocks, they differed in their mode of conception. That is, instead of stressing a block of the model as a piece of matter occupying an abstract three dimensional space (a geometric body) which contrasts with the flat structure of a figure, all the PROEM's students and the majority of the manual workers compared a physical body occupying real space with an abstract plane surface.

Thus, when I asked a PROEM's student if he would recognise any correlation between the on-screen figures and those presented in the model, he observed:

Their contours may be alike but they are different because these are drawn on the computer while those buildings are real. Let me explain it better ... these are merely contours ... so, they are not real while those exist in the middle of Brasilia they are really real ... people can enter and get out ... people can hide from the rain ... people can sleep inside them because they have walls and roofs while these are only lines without a body.

Explicit in the statement above and implicitly voiced by another PROEM's student is the view of a pictorial figure as an absolute two-dimensional enclosed plane whereas a block has a three-dimensional cubic space which can be entered.

They are different because those blocks were in real space while these blocks are hidden on paper. I mean, we can't see the windows or the doors nor the people inside it ... but we can say that their walls have similar contours.

In my opinion, this possibility of 'walking through' mentioned by PROEM's student marks a difference from the abstract spatial structure suggested by the secondary students. That is, while the first group emphasised an empirical visual experience the other group stressed an intellectual concept of space.


Like the PROEM's students, most of the manual workers observed the distinction between a plane figure and a solid block as stressing a tangible existence in contrast with the 'reduced' form of the pictorial figure. However,

again, I noticed differences in the graphic designers' comments. For instance, the designer Luis said that "these figures are only a drawing ... if we want to see them like those buildings, I mean, as solid, we would have to assemble these plane figures and only then would they become a solid". This observation insists on the possibility of creating volume by relating various planes. This connection between plane and volume was not mentioned by other manual workers or by the secondary students who saw the blocks as well as the figures as an entity, or better, as a whole structure.

The graphic designer Mario recognised the correspondence between the figures and the blocks viewed in the model as did other workers. However, he was more emphatic than other workers in relating them by a likeness in their structure:

Although their dimension is not exactly equal it is perfectly possible to relate these figures with those viewed in the model. Thus if you remove, for example, the ministry block there will be a mark on the floor exactly as shown. That is, the block occupies a space equivalent to this rectangle on the plane. The same applies to the others except the hemispheres. In this case, the hemisphere turned down will appear a trace while on the other there will be a imaginary point and the projection of the hemisphere. If we show it to an engineer he will be able to read it.

Through observing this 'polysemy' constructed by different groups in this activity, I am drawn towards a conclusion. My empirical material reveals that a pictorial figure does not 'mirror the structure of concept' as suggested by for example Von Glasersfeld (1987) and Fischbein (1993), or as claimed by Janvier et. al to whom "certain representations [in particular geometrical ones] are so closely associated to a concept that it is hard to see how the concept can be conceived without them" (Janvier et. al 1987:110).

Semiotically speaking, my data support the idea that the signified is not "coupled" to the signifier. Thus, when the participants saw on the computer screen a set of figures, there was not an analogous bond between, for instance, the pictogram  and the 'concept' square, the anticipated correlation if we believe in the iconicity of a pictogram. In fact, as Hall (1980) reminds us, single visual signs [such as the pictorial figures] "may, of course, be so widely distributed in

a specific language community or culture, and be learned at so early an age, that they appear not to be constructed (...) but to be 'naturally' given" (Hall 1980:132). However, as Hall goes on to explain, even apparently having an universal meaning, the "visual codes are cultural specific" (ibid. 132). Indeed, the plurality of responses noticed in my research indicates that even stable visual properties, such as pictorial figures, are "cultural and variable" rather than "textual and fixed" (Bennett and Woollacott 1987:81).

When those "cultural and variable" aspects are neglected, elements like shapes and forms become correlations encapsulated within a formal system independent of an actual situation of significance as I shall discuss next.

7.2.2 The 'equivocal of the referent'

This activity consisted of discussing the model of Brasilia represented by a screen image in which the figures equivalent to the shapes of the buildings were arranged in the same way as they might appear in the real setting (cf. figure 3.4 page 73). My focus of attention here was to examine what the diagram evoked in the three different groups and how this might be explained.

When I asked the secondary students what they saw and what they could say about the computerised diagram, some of them associated it with familiar objects, as the examples below illustrate.

Carlos: I see a flight deck for the take off and landing of aircraft. There is a central track and besides it there are various rooms where the aircrafts are kept.

Ana: I'm not sure exactly what it is. It seems to me to be a view of a classroom where there are many identical tables, set in a parallel row and a teacher's desk is placed in front of it.

Alex: It may be an aerial view of a research room - a library with many tables placed in a symmetrical order.

In contrast to the above statements, the secondary students Elisa and Marcelo had no doubt that the diagram represented the central area of Brasilia viewed from a

plan point of view. The first one said: "I see an aerial view of Brasilia", the second remarked: "It is evident that it is the Esplanade area shown from a plane view". In a similar way, the graphic designers Mario and Luis agreed with these two students. Thus, one said: "This is an outline of the Esplanade. I would recognise this in any situation even if I didn't draw it as I once did, I wouldn't have any doubt about it"; and the other observed: "I clearly see a graphic reproduction of the Esplanade".

Based on the discussion in Chapter 2, we can analyse the above statements from two perspectives. The first way of understanding that connection - the diagram versus the interpretations - is to view the diagram as a closed system of signs; the observer creates or abstracts meaning from the various clues and evidence provided by the figural features of the diagram.

The observer is thus defined as an active agent who relates or recounts what she/he saw inscribed within the text. If a visual text is considered as a motivated sign, that is, 'it possesses some of the properties of the thing represented', it is feasible to say that the above statements made by secondary students and designers were based on the internal logic of the diagram which resembled a familiar scene, that is, the signifieds were motivated by the referent³. Specifically, when those persons associated the diagram with a 'research room', 'a classroom' or as 'the central area of Brasilia', they knew that the diagram did not resemble those scenes at all in looks but the diagram included similarity of abstract relations or structural homologies which allowed the making of such a mental connection. This rational understanding of a diagram is a central concern of those researchers who have taken Piagetian constructivism as their framework of reference. What is here observed and examined is the mental act of seeing by which the subject acts upon the objects in order to grasp meaning.

³ In this formulation, as Barthes points out, "everything happens as if the picture is *naturally* conjured up the concept, as if the signifier gave a *foundation to the signified* (Barthes 1972:129-30).

From a contrasting view, the socio-semiotical approach is not concerned with analytical abstractions by which the subject extracts meaning from a visual text but rather seeks the social signification that underlines their way of describing what they saw. If we analyse those comments made by secondary students and designers from this point of view, we may suppose that what enabled them to recognise the diagram as a representation of the central area of Brasilia or "as a research room", was their social familiarisation with a set of graphic conventions and visual codes. These express spatial properties such as shape, position and direction and these, in my view, formed their frame of reference. In other words, we can say that a diagram (as a visual text) may look like objects in the real world because it reproduces the 'condition of perception' in the viewer. However, the condition of perception is selected through codes of recognition which are culturally derived.

My data support these suppositions. For instance, if we compare those secondary students who did not recognise the diagram as a graphic representation of the model with the observations made by PROEM's students, it is possible to say that even these students showed more familiarity with the notations of plane projection than PROEM's students as the examples below demonstrate:

Kelvin: This is a very strange drawing. I feel uncertain about it because at first sight I thought that those two circular pieces might be the door of the building but it is odd to have a circular door, right? I really don't know what figure this is because it seems very complicated - meaningless - to imitate something.

Maria: This is a strange figure. It seems like a thermos bottle viewed from the inside; I see those sides as if it were round and made by glass exactly as when we look inside a thermos bottle. Also there is a 'mouth' through which the coffee gets out. This is the only possibility for understanding this drawing made by the computer. I don't what intention he [the computer] had when he drew it.

Gil: I've never seen this picture before .. thus it is difficult to say what it is ... maybe it is a 'rocket' that took the man to the moon.

Similarly, the manual workers manifested difficulties in describing the diagram viewed on the screen. Thus the certainty or obvious intelligibility expressed by the designers Mario and Luis was not apparent in the other workers' descriptions. For instance, the carpenter Sebastian observed that "if someone came to my

workshop showing me this design, I wouldn't know what kind of piece he was asking me to build. I would need more details about it in order to make the piece". Another carpenter noticed: "it seems like a type of project ... but it is difficult to say what it is". In justifying his difficulty, Sebastian suggested that the absence of the three dimensionality inhibited his recognition of the diagram. This was expressed in the following way: "when a client shows me something drawn on paper it is shown in a solid way. But, in this case it has a flat shape - without a real body - thus it is more difficult to know exactly what it is". From socio-semiotical perspective, we would not say that this difficulty to recognise objects in a diagram represented an incapacity to deal with an 'abstract picture'. Rather, explicit in his comment is that through his activity as a carpenter, he was more familiar of seeing objects expressed through three dimensional conventions rather than through two dimensional images, difficulting thus his interpretation of the diagram.

What this point of view suggests is that there are no direct impressions or perceptions. This is because what we perceive is always a sign of the thing rather than the thing itself which makes the notion of reference problematic. Thus, the sign as a 'refraction' of language, culture, background, education, our social frame of reference, exerts a great pressure on the selection of perception as the observation made by carpenter Sebastian seems to indicate.

7.2.3 A transitory correlation between signifier and signified

In this activity, I used the computer's graphic capability to create three different representations of the model (cf. figs. 3.5, 3.6, and 3.7 page 73-74). When I designed this activity, I expected that these distinct means of expression would present new readings, revealing different connections between the signifier and signified.

In fact, if we compare the responses given by the different groups in relation to the model of Brasilia represented in a three-dimensional form with

the present activity it is possible to notice differences. For example, whereas all the participants, without exception, recognised the three-dimensional model as a graphic representation of the central area of Brasilia, this time the majority of them did not identify the first representation depicted on the computer as part of the model. Indeed, all the persons, except one graphic designer, only acknowledged the model when they saw the third view.

Looking at the three representations shown, one may suggest that those results are linked to the differences between the ways which the model - or parts of it - were depicted on the screen. That is, although all of the representations were modelled in a schematic way - with a wired structure - one could say that the first two views (figs. 3.5 and 3.6) were more abstract than the third one (fig.3.7) which was more realistically depicted. If we accept this opposition of the abstract against the realist, we are suggesting that one is opaque and ambiguous whereas the other appears to be more 'naturalist' and is therefore more easily recognisable.

I shall avoid these terms in my analysis for two main reasons. First, the opposition between an abstract and realist view is problematic from this research's theoretical point of view, since the notion of a realist visual text suggests a 'transparent representation of the 'real' - it would suppose that it was possible to have unproblematic access to the referent since all the participants were familiarised with the central area of Brasilia; a possibility rejected by the social-semiotic approach. Second, when I designed the activity on the computer my intention was not to use its dynamic features in order to find out whether one view was more abstract than others. Rather, my interest was linked with a fundamental concern of the semiotic approach. That is, the assumption that "different discourses are mobilised in different ways by different viewers and in different contexts" (Buckingham 1993:18).

Building on this, I believed that a visual text within a medium had the ability to transform and signify, rather than to simply transport and recall - the

images might provoke new interpretations and concepts. The power of the graphic computer lies to some extent in its capacity to modify, dismantle and distort the images, besides its capacity to view any image - as whole or a particular aspect - from several positions and orientations.

Using these resources, I deliberately chose a way of depicting the model which was graphically ambiguous. I wanted to break down the logical and rational form of the model under which the persons normally viewed it, by presenting the model in an unconventional way. Thus, instead of showing the blocks clearly differentiated from each other in the symmetrical and ordered space which they occupy, I distorted and dismantled, producing a different appearance. By also showing the blocks in a different position and orientation, I wanted to observe how this unfamiliarity would be interpreted by different groups.

In fact, the effect of dislocating the blocks from the context of the model and the effect of overlapping the flat planes of the blocks (fig.3.5) did indeed suggest new meanings. For instance, when looking at the blocks depicted on the screen, all secondary students associated it with a pyramid, a description not mentioned in other contexts. The following observations illustrate this tendency. One student described it as:

A set of flattened parallelepipeds. One of its faces would be a rectangle and all of them have the same height except the lower bases which gradually get smaller, it seems a pyramid in which the central area has a polygon base.

Another description was: "this figure gives an idea of deepness; it also may be a truncated pyramid of which the plane view is given".

This geometrical quality was not mentioned by other groups to whom the figure was associated with a tunnel or a corridor, an attribute not referred to when they saw the blocks depicted in the three dimensional mode.

Gil, a PROEM's student: This figure is like those tunnels which are shown on the TV, I mean, those tunnels which appear on the police films where the policeman chase the gangster; but he cannot escape because in the end of the tunnel will be many cars full of men, a trap; all of the men are equipped with heavy machine guns.

Maria, a PROEM's student: This figure reminds me of a corridor that there is under the bus station. It gives us a sensation that we are getting in it. It also something we see on the TV but I can't remember exactly what it is.

The carpenter Raymond: I see a pile of rectangles ... they are not of the same size rather they diminish when moving away from our eyes. It looks like a tunnel.

The potter Tom: This is a tunnel which we can enter. Looking out through the tunnel we see that there is neither any object nor people at the end.

The first conclusion that can be drawn from the data is that if we play with the figures or objects, exploring their 'multi-accentual' quality, that is, the openness to being dislocated, disjointed, and displayed, it is possible to observe that the sign - as category of meaning - changes and moves and is sensible to new types of expression.

In the present activity, we can say that the production of different meanings was provoked by two main reasons. First, the computer with its specific levels of codes and operations provided a dynamical way of representing the blocks which would be difficult to visualise from a physical model. That is, the computer allowed us to construct a visual representation whereby the blocks lost their wholeness and clarity. Thus new images and concepts of their characteristics and properties were generated. In this sense, we may say that the production of different meanings was an effect of certain computer mechanisms which allowed a play of signification not present in other contexts. Second, if the point of concern is not imagistic but social, we may say that it is not a coincidence that all secondary students described the figure as a geometrical figure. In my view, a 'code of recognition' comes to them from their social milieu: codes of the formal education system. In saying this, I am arguing that the viewer's relation to the screen is a social rather than a mental act and the subject acts upon the objects in order to take meaning.

As occurred in the previous representation, none of the participants, except one graphic designer, recognised the second view depicted on the screen as the model of Brasilia. Rather, the manual workers and the PROEM's associated it

with other everyday scenes or objects, while the secondary students simply gave a procedural description of the structure of the drawing.

Kelvin, a PROEM's student: This is like an empty terrain that there is close to my house where there are many pieces of iron, pots, and some strings thrown all over. I also see a box-like a square placed in front of those irons.

Maria, a PROEM's student: Right now I see a lot of squares in which all of them are positioned in a similar manner to which we play cards. Let me explain when we play cards this is scattered on the table, right? So, these squares are in the same position as the cards.

David, a potter: I see many open boxes touching one another. The first box - on the left side and closer to us is the biggest one; the others get smaller while they go away from our eyes.

Raymond, a carpenter: These seem like a skeleton of many buildings. I mean, they are only contours, without showing their real body.

Elisa, a secondary student: I see many rectangular structures. Their positions are perpendicular and parallel having these right angles, here and here.

Alex, a secondary student: I see rows of horizontal and vertical lines at 45° angles.

Although all participants recognised the third view as a graphic representation of the model of Brasilia, all of them noticed a certain strangeness in the way it was displayed on the screen. Marcelo, a secondary student, observed that:

I see a bizarre Esplanade. I mean, in this aerial view the position of the blocks does not correspond to the reality since these blocks are not arranged in a symmetrical way as those of the Esplanade. That is, they are neither perpendicular nor parallel but they are inclined.

Such strangeness was noticed by a PROEM's student in the following way:

I see many sketches of buildings which are twisted and lopsided. It seems to me that someone - a very strong person - pushed them down.

According to a manual worker:

These blocks might be the blocks of the Esplanade but in reality when I cross that place I don't see them like this. I mean it is not possible to see those blocks in the way it is shown on the computer. I mean, it is not thinkable to see it in reality unless we imagine that we are flying.

Notice that none of the responses showed any inclination to 'understand' the computer's point of view; to take as given the meaning of the objects and see how it had been apparently manipulated by the computer. Thus I can conclude this section by saying that the socio-semiotical approach teaches us to be suspicious

of arguments that only take a concept, geometrical ones included, as an essential core. My empirical data shows clearly how a range of representations can produce differences of meaning, how semantic values vary according to coding rules each time by the context, what Eco calls "the mobility of semantic space".

7.2.4 'The mobility of semantic space'

After examining the way the individuals described and interpreted the four activities, I may say that in the fourth activity, where a set of three-dimensional figures were displayed on the screen (cf. page 75), there was an important difference in comparison to others both on the computer and in other settings, mainly as far the manual workers and secondary students were concerned. More specifically, while in other settings, particularly the wooden blocks, the manual workers connected or interacted with the elements as if they were moulding or shaping a solid block, in this activity they tended to make relations in which the focus was on the decomposition of the figures. In terms of our categories of analysis, we may say that here they tended to characterise the figures through 'decomposition' rather than through 'constructive description'. In relation to the secondary students, we also notice differences between the observations they made in this activity with those made in other contexts. First, while in other activities and settings the secondary students tended to polarise the elements, in this activity they connected them. Second, instead of seeing the objects as a whole unity as before, in this activity they decomposed the whole given figure into parts and combined these parts to make another figure. Another relevant aspect pointed out by the same group was the 'strangeness' of the figures. The majority of them complained of their lack of familiarity with the position of the figures on the screen.

In what follows I shall analyse how the secondary students and the manual workers connected and decomposed the figures.

After describing several figures present in the setting, the student Marcelo made the following observation: "since there is the possibility of moving and enlarging these figures I may say that we can make innumerable combinations". After saying this, he looked at figure 3.8 (nº 9) observing: "if we move those cubes at a frontal view and then change them into various sizes, we would see something like this":

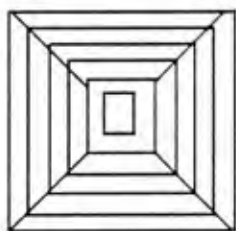


Figure 7.1 Cubes at a frontal view

Marcelo commented on his drawing: "This shrinking of the squares doesn't affect the properties of the figure. The only difference is that the radius gets smaller if we have the foreground square as the point of reference". He also pointed out that this set, composed by different sides of squares might be a "truncated pyramid, similar to that one you showed to me, with its apex facing us or as a truncated pyramid with its apex directed away from us".

The student Alex also moved and created new figures below [figures 1 and 2] see 7.2. In relation to figure 1, he observed: "I built a set of six cubes. These cubes are in touch at one point, in fact, there are two points in contact and one can be spun without disconnecting these two points". In relation to figure 3.8 (nº 2), Marcelo pointed out: "here I created three sets of parallelepipeds. The one on the left is in touch by one point. The second has more than one point in contact and the third is penetrating one into another since the planes intersect one another".

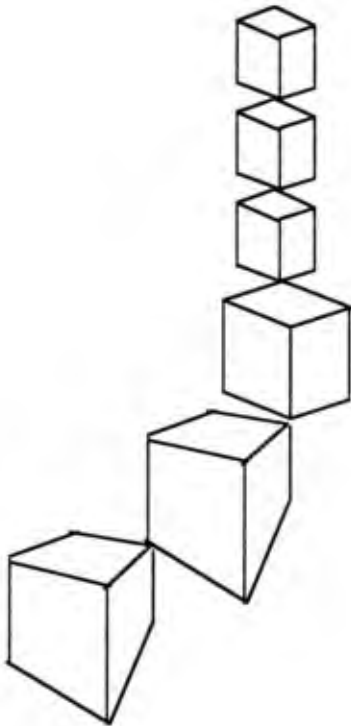


Figure 1

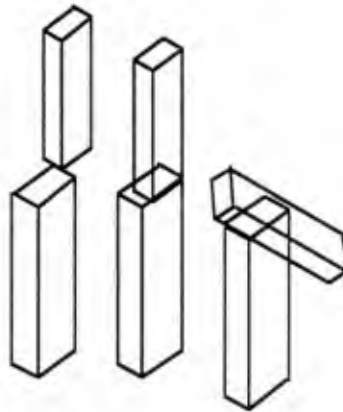


Figure 2

Figure 7.2 Linking cubes and intersecting cuboids

Observing the connections made by students Marcelo and Alex, we can say that when they transferred the surfaces of the figures they created something new. Semiotically speaking, we might say that they created new sign-functions. That is, the correlation between the signifier and signified, in this activity, involved a different quality as compared to the other activities on the computer and to the other settings. The most visible difference is that this activity elicited a tendency not exhibited in all the other contexts, to establish connections between the elements rather than to polarise or to isolate or to look at the blocks from a fixed point, a single station, without moving. Thus, instead of saying; "this is totally different from that", the discourse here was: 'a set of different squares might be a truncated pyramid'; or a set of parallelepipeds can be "in touch by one point" or inter- penetrating "one into since the planes intersect one another".

These created and manipulated relationships and transformations cannot be separated from the context in which their production occurred. Rather, this

difference, in my opinion, arose from the possibility of exploring shapes and forms in a more dynamic manner than possible in the other activities and settings⁴. That is, in this activity there was the opportunity to create, rotate, move, and enlarge the figures not available in other contexts, motivating thus new 'readings'.

Similarly, if we examine the connections made by the manual workers within the present activity, it is possible to say again that new concepts and meanings about the geometrical figures were constructed. For instance, while in other settings, i.e; the wooden blocks, the manual workers connected or interacted with the elements as if they were moulding or shaping a solid block, in the present activity they tended to make relations in which the focus was on the decomposition of the figures. Thus, they compared the external and internal parts of the figures in which the figures were cut or divided into cross-sections. This tendency may be illustrated through the following comments.

Looking at figure 3.8 (nº 8 and 10 page 75) portrayed on the screen, the graphic designer Luis observed that they were different from an external point of view but if one looked at those blocks in an interior way it was possible to say that there was a point of intersection between them. He explained it with the following words:

I don't know how to explain it to you but if one examines those entire blocks or smaller parts of them you are going to see that they can be alike ... not from their outside but from the inside. Let's take an example: look at the block 8 and 10 on the screen, it is clear by its appearance that the block 10 is bigger than the 8, however, the block 8 may be contained in it. Let's say that this block 10 is a building in which you divide into parts ... we are going to see that the block 8 may be inserted in the block 10. I also may say that if we cut out the block 8 in various parts we could have many little dices.

In attempting to be more explicit in his description the Luis drew this figure:

⁴ Certainly this dynamic way presented in this activity could be available in the other computer activities. However, when I created the research design of my study, the intention was to explore the power of the graphic computer in two different ways: that is, while in the model of Brasilia I used its capacity to modify, dismantle and distort the form of the model under which the persons normally viewed it; in the set of three dimensional figures I wanted to encourage the participants to enlarge, reduce, rotate and move the figures.

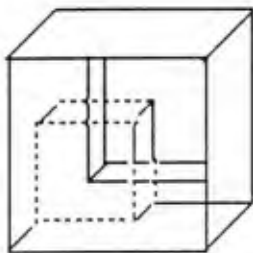


Figure 7.3 An embedded cube

The designer Mario, like Luis, also indicated the possibility of seeing sections in the blocks. He demonstrated it through the following illustration.

Look at that figure [3.8 n° 6] we see that its surfaces fit into another. In fact, we might say that instead of being, as apparently seems, two cubes, we might see the two as a whole in which the interior surface would be a cross-section. Thus, the way the figure was displayed allows the observer to look at the interior, and, likewise, the exterior.

Indeed, Mario made various comments about figure 3.8 (n° 6) in relation to the possible ways of looking at it through its sections. He observed that:

If one cuts off a wooden block like those you showed, the surface that arises from it is a cross-section. Looking at that figure it seems empty, in fact there is a vacuum inside the bounded surface but we can imagine surfaces which cross it. Looking at this figure right now, some surfaces come to my mind, for example, I can picture a straight cross-section surface that may be inclined or a hexagon plane.

The drawing below illustrates his description.

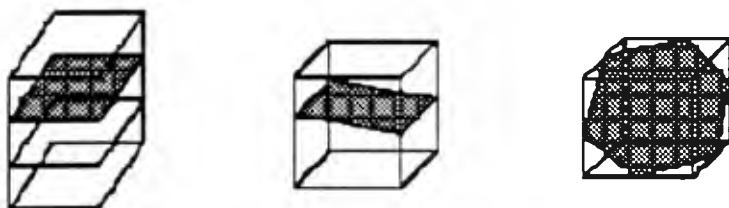


Figure 7.4 The sections of a cube and a cuboid

In relation to the hemisphere and the circle, Mario noticed that both of them were closely linked because any section of a sphere is a circle. In relation to the hemisphere he observed "as I mentioned before it is an orbiting solid, thus a semi-circle in space; but the semi-circle rotates only half turn while in the

sphere it is all the way round". After saying this, Mario explained how to assemble a sphere:

We can take various circumferences of diverse sizes and tie them together, one by one, so that all the circumferences are joined, moulding a spherical surface. Also you might tie some circular wires of different sizes in like manner a sphere will appear.

Looking at the observations made by manual workers and the secondary students, we notice that under certain conditions, in particular contexts, a visual text tends to be 'read' in a particular way. Here, the dynamic mode of addressing the forms and shapes pointed to new vision of the relation between signifier and signified, generating thus connections and disconnections, new modes of interpreting visual texts.

However, this did not occur with the PROEM's students who were told of the opportunity of manipulating the figures viewed on the screen. Most of them replied saying that they would not like to do it because they might damage the computer. Although I tried to convince them that this was a remote possibility, most of them resisted touching the mouse. Their apprehension of touching the mouse may be summarised in the follow words:

I don't know but my hands are very tough to touch it because this computer seems very fragile; besides I have doubts if I can say or do something with it. To be honest with you, it is like a black box to me in which I don't have any idea how it works or what it does.

Therefore, as with other activities and settings, the PROEM's students continued to describe the elements depicted on the screen in terms of everyday life objects and the influence of the computer was held at a distance. For instance, some students noticed: "these blocks are like bricks in the space" or "these boxes of chocolate are flying". Yet, "the computer put these blocks inside a container in which these blocks were shaken and from which they were thrown".

What is important to stress here is that the computer, at least for those participants, who were happy to touch it motivated readings about the geometrical figures not present in the other contexts. In my view, this difference indicates a fundamental issue: meaning changes with the situation, and the changes

occurring in the computer context were related to its specificity, particularly its dynamic mode of producing images and its unique modalities of enunciation which allowed different articulations.

In what follows, I shall examine how a set of figures represented in an unconventional way became intricate and obscure for the secondary students. All of them declared their unfamiliarity with the position which the nº 3,6,9 and 10 [figure 3.8] were portrayed on the screen. This strangeness was mentioned by the student Carlos with the following words:

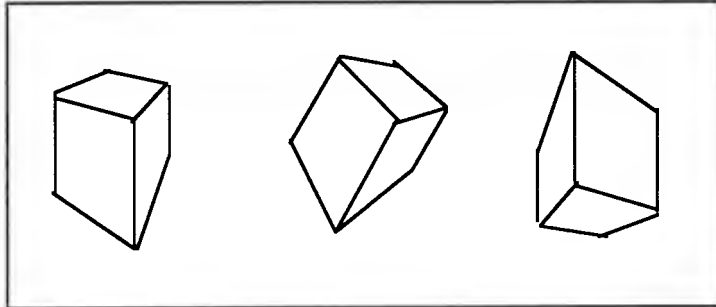
On the computer the objects gained another dimension. It is different when one looks at these figures presented in books. I mean, these blocks shown on the computer seem strange and odd because if you look at a parallelepiped drawn in a book it would be in a plane and generally it stands up. Well, they are in a plane but they seem more deep. I don't know how to explain it ... but on the computer the figures seem more dynamic maybe because they are shown in an oblique perspective.

Justifying his lack of familiarity with the position of the objects Carlos said: "I'm not sure if this figure [he was referring to nº 10 fig.3.8] is a cube or a trapezoid, but I know it is not a parallelepiped because it does not have right angles".

In relation to nº10, another student noticed: "you forget to make the dotted lines. I mean, usually in the textbooks this figure is shown with dotted lines, so all its faces can be seen". He then regarded nº 6 and 9 [fig.3.8] "often the figures shown in texts are separated one from another. That is, it is difficult to see the insertion of one figure into another. Normally, they are shown separately". Also, he observed that "the position of the figures is often presented on the horizontal position. Lastly, Marcelo stated "it is quite bizarre to see a cube stretched out like that one".

What these above comments indicate is how a figure or a set of figures represented in an unconventional way became intricate and obscure for certain people, such as the secondary student Carlos and Marcelo or Ana who made comparable observations in relation to the nº3 [fig.3.8]: "I'm quite confused about this figure. I don't know exactly what it is because it's not obvious that it is

a cube". At that moment I reminded her that if she wanted she could rotate, move, and enlarge the figure. I did this in order to find out if it would make any difference. She created the below set of figures:



7.5 Rotating the cuboids

After rotating the figure Ana stated "I am still baffled about this figure". In a similar way, another secondary student observed:

I really don't know what these figures are neither their names. It seems a cube stretched out in which there is a prolongation in one of its edges and the angles are not right angles. However the effect of moving the block makes a difference I mean the size and the viewing point change [in attempting to illustrate this last observation he created the below set of figures].

What the above observations indicate is that if we 'play' with the figures, modifying their positions and orientations, the formal school students experienced difficulty in identifying figures when they are presented in non-standard orientation.

Alex, another student, also observed the position of the blocks: "What attracts my attention is that these parallelepipeds are shown in different perspectives and some of them appear to pass though the air, that is, flying". After saying this, Alex remarked that "if we want to see these parallelepipeds grounded on a plane it would be necessary to trace a horizontal line ... crossing through out the screen". Alex created the following drawing:

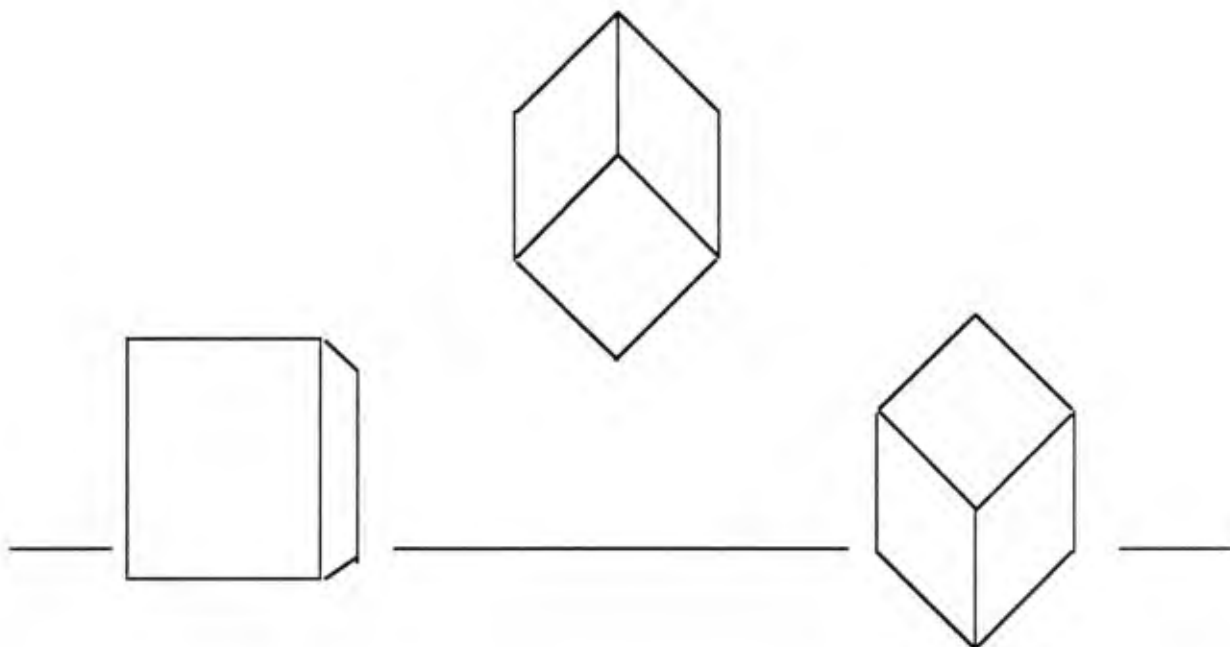


Figure 7.6 Parallelepipeds shown in different perspectives

After creating this set of figures, Alex fixed his attention on the figure displayed in the middle saying:

It seems like a trapezoid ...but it depends on the angle we look at it ... I mean, if we fix our vision on that vertices we might see a cuboid, that is, in this particular angle you might approximately see a complete cuboid since the internal edges of the figure are hidden ... but if we look at this figure from the frontal view we would see a tetrahedron which has three faces.

While Alex observed the majority of the figures depicted in the setting without any manifestations of strangeness, the same did not occur with the nº 3 [fig.3.8]. When he looked at this he observed: "Honestly it is difficult to describe this figure ... Does it have a specific name? It is some kind of prism". Another student said: "this figure is quite strange ... I know it is not a cube. I mean, it seems like a tetrahedron instead of a cube".

In analysing the above comments it is possible to observe how by manipulating the viewpoint, the 'obvious' meanings derived from social and cultural experiences become destabilised although the secondary students still tend to impose a formal (but new) meaning on to what they saw. In other words, we can say that the context analysed here - the activities through the computer - provided a way of representing objects and their relations in which the students

could bring a different level of abstraction, that is, new meanings of conceiving the formal identity of the geometrical elements. In this way, a cuboidal or a cylindrical form gained a different meaning when compared with what was said about these forms/shapes in other contexts [settings]. That is, the "strangeness" of the figures provoked commentaries which did not appear in other contexts. Yet, their strong tendency to polarise the blocks as had occurred in other contexts was inhibited by a model of communication which insisted on an enquiry into a dynamic of production.

These new forms of expressions constructed by secondary students demonstrate that the signs are 'the provisional results of coding rules which establish transitory correlations of elements' thus semantics values vary according to coding rules given each time by context. However, as Bakhtin reminds us, the 'situational variables produce differences within the field of interpretations, but the limits of that field are determined at a deeper level, at the level of what language/codes people have available to them - which is not fundamentally changed by differences of situations'.

8 An analysis of the free drawings

The aim of this chapter is to examine how the participants gave meaning to geometrical forms and shapes through their free drawings. After discussing different sorts of contexts in which various sets of three-dimensional elements and pictorial figures of different forms and sizes were presented, I asked the participants to make a drawing utilising these elements and figures. The intention of the drawings was to observe how the geometrical figures and objects would be conceptualised through a graphic representation.

The drawings will be analysed in the light of one of the main analytical tools discussed in Chapter 2: the concept of visual sign. One central and persistent assumption which arises from this theoretical concept is that images, as well as words, carry cultural meanings. By taking this supposition as the main guide for 'reading' the drawings made by different people, one important task is to discover how cultural signification is expressed in their drawings.

A useful way to access such modes of signification is to analyse the drawings by correlating two things: the meaning of the signifier and the social motivation of the producer of the sign. In other words, it is important to analyse, simultaneously, how the geometrical images were constructed and how the formal and non-formal schooling influenced the way in which the geometrical images were represented in the drawings.

In attempting to analyse this dual point of observation, I start by looking at two drawings produced by a secondary student, Alex (see next page).

8.1 A formal relationship between signifier and signified

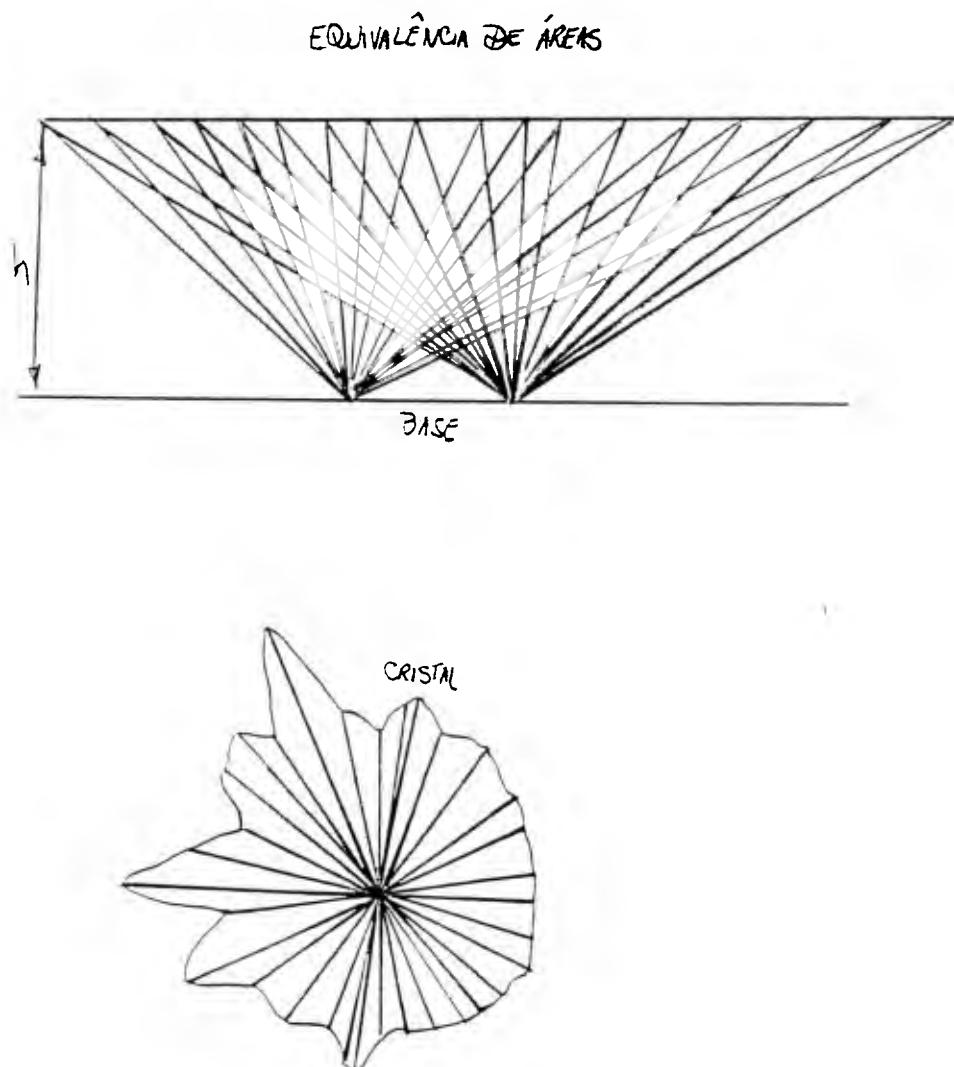


Figure 8.1 "Equivalence of areas" and "Crystal"

Looking at both drawings, we can say that both were not based on anything that exists in the tangible, physical world. Rather, in order to express his idea of geometrical elements, the student Alex was inspired by geometrical reasoning. More specifically, his drawings depict abstract configurations which represent, diagrammatically, geometric concepts. "This drawing", as Alex commented about the first drawing, "represents the principle of the triangles of the same area because it has the same base and same height". This drawing was called by him

"Equivalence of areas". In relation to the second drawing, called "Crystal", he associated it with chemistry saying that "organic chemistry has a strong link with spatial geometry ... the example of a crystal demonstrates this since it has a cubical structure".

If these drawings were analysed by logico-analytical perspective, the abstract and linear way of depicting a geometrical elements might be interpreted as an ability to abstract. This ability would be demonstrated by his mental capacity to represent and interpret a geometrical object. Analysing these drawings from the socio-semiotical point of view, we might say that the signifier (the drawing) and the signified (the meaning) constructed by Alex were clearly influenced by his social experience with formal academic geometry. More specifically, living in a pedagogical context in which geometrical objects are primarily defined as mental entities which are not supposed to possess any actual reality, supposedly ideally perfect, it seems not accidental to identify these particular patterns of viewing in Alex's drawings. This influence may be revealed by the emphasis on the abstract. Equally prominent are the other aspects presented in his drawing such as the uniform and symmetrical way of organising the lines and planes.

In what follows, I shall reinforce the point that the specificity of the social practices may effect the way people construct visual signs. In order to do this, I shall contrast two drawings: one made by a secondary student and another made by a potter.

8.2 A formal and non-formal way of viewing and relating forms

Elisa, a secondary student, preferred to depict the geometrical forms as an actual element of everyday life rather than as abstract entities as Alex had done. She sketched the area of the Esplanade as the figure below illustrates.

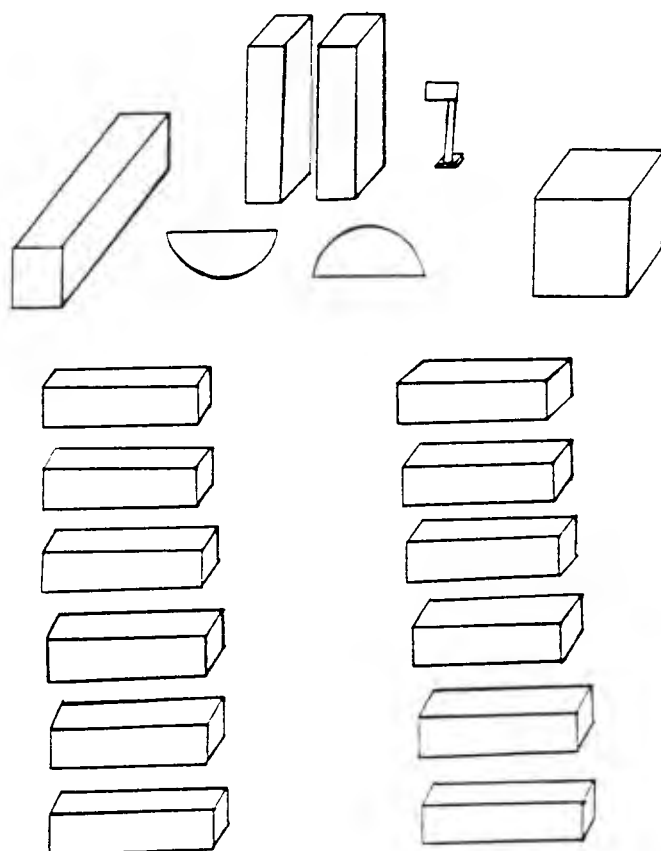


Figure 8.2 A formal model of visibility

By observing the arrangement of the blocks in the picture we notice that each block remains self contained and unambiguously depicted; that is, the blocks are clearly differentiated from each other without any overlapping. Suggesting an orderly, clear-cut and rational space. Here, again, it is possible to observe the formal model of visibility. That is, the signifier constructed by Elisa expresses formal and logical qualities. From the socio-semiotical perspective, these characteristics selected by Elisa symbolise her interaction with an educational practice in which spatial and geometrical objects are analysed as a set of logical and formal entities.

Lets now observe the drawing made by the potter Tom.

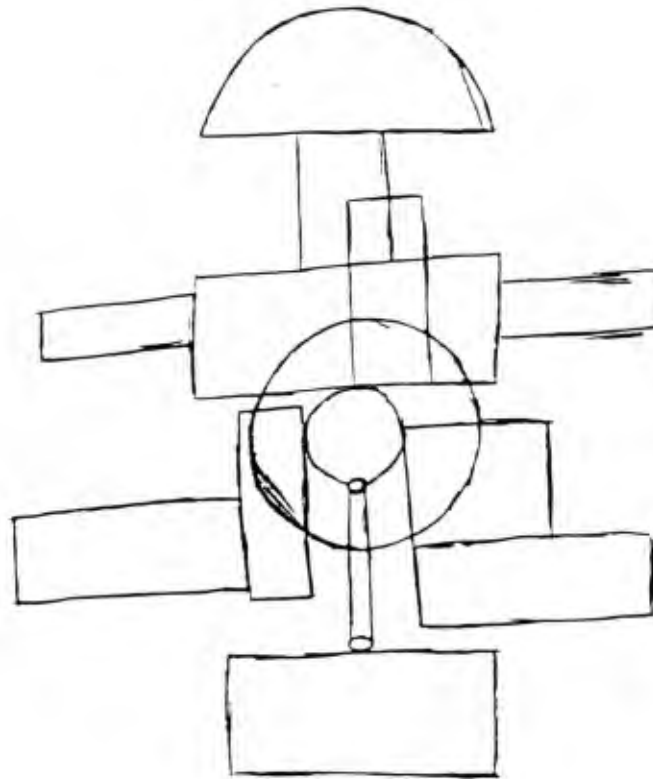


Figure 8.3 An informal model of visuality

Sticking together bits and pieces of geometrical shapes, he drew his own view about the model of Brasilia. Looking at his drawing, we clearly note that Tom was not primarily interested, as Elisa was, in depicting the model as it corresponded to reality - that is, with its order, cohesion and regularity. Abandoning its coherent spatial organisation or its sense of unity and completeness, the potter Tom framed the model of Brasilia so that its parts symbolised a fragmented pioneering workmen - a *candango* - who worked on Brasilia's construction.

Tom described his drawing with strong subjective reference:

I have tried to outline a human figure playing with those objects and forms. The figure that I have in mind is a northeastern who links all the forms of Brasilia. I mean, we can see a broken *candango* wearing a typical northeastern hat. His arms are open which means that he still has some hopes despite his stomach is empty as that circle in the figure shows. In my opinion, a northeastern's figure should be the emblem of Brasilia.

Analysing the geometrical components present in the picture made by the potter Tom, it is possible to recognise that they are neither isolated nor unarticulated. Rather, he created a two-dimensional *collage* where various planar surfaces are interconnected: they are either overlapping or are continuous. If we understand a visual image not as representation of a mental concept but as a product of our interaction with the real world, we might say that his way of constructing and interpreting a geometrical signifier carries echoes of his social experience with geometrical objects. More specifically, that *collage* of planes presented in Tom's drawing may express the network of shapes and forms present in his way of constructing pots. As he observed: "whenever you press the clay in your hands the shape of it changes ... sometimes I do a square other times I make it like a ball or egg".

Like Tom, the graphic designer Mario, who also lacked the formal and logic of school geometry, showed through his drawing a non-formal way of viewing and relating forms and shapes (see below).

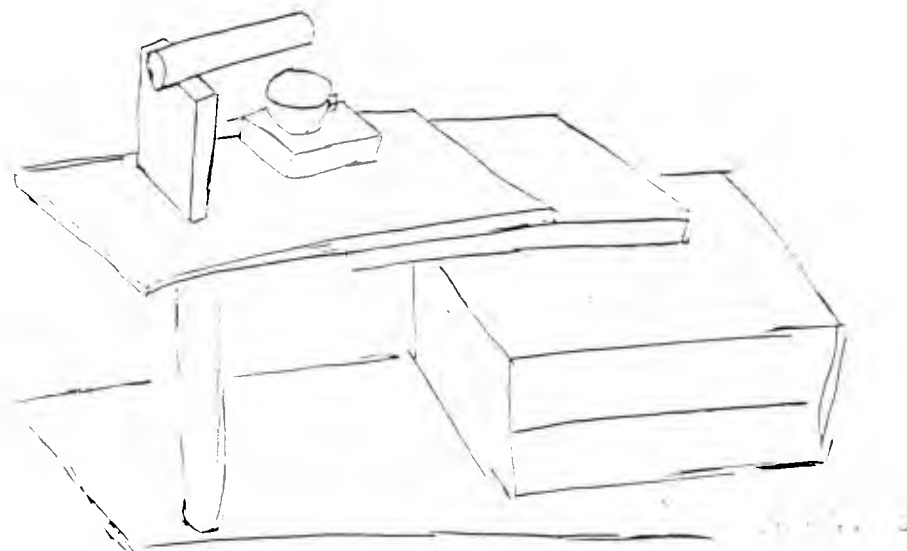


Figure 8.4 Interconnecting different shapes

Mario drew a table consisting of a straight flat top supported on two legs. The legs were outlined in an asymmetrical way: the one on the left side had a cylindrical form while the other on the right was composed of three different rectilinear overlapping surfaces. On the table are exhibited four rectilinear, cylindrical, and spherical bodies. Some of them suggested a sort of equilibrium in relation to each other, like the cup on the top of the box, whereas the other two seemed unbalanced in connection to each other.

Contrasting Mario's drawing with the secondary students' drawings, we might say that while in the former group the separation prevails, in Mario's design the elements have different spatial interconnections: the elements are adjacent to one another, or they may be interconnected, or they may support each other.

8.3 Dynamic versus static

The drawing made by Luis, another graphic designer, expressed something different from the other workers:

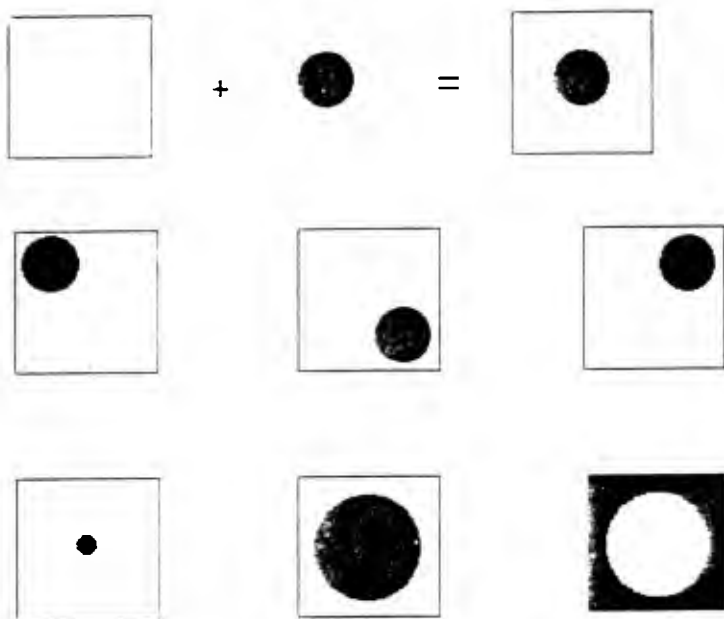


Figure 8.5 A dynamic view

First, instead of using pencil and paper he asked me if he could draw on the computer (using Mac Draw). Second, rather than depicting an everyday scene or object, as the other manual workers had done, he preferred to play with flat squares and circles which gives his drawing a more 'formal' quality. However, this 'formal' quality is different from that of the student Alex who also drew figures on the paper. More specifically, while figures presented on Alex's drawing represented a static mathematical construction, in Luis's drawing the figures represented a kind of game. In this, movement was the most remarked characteristic as the planes were added and subtracted. By adding and subtracting planes, he introduced mobility and dynamics into the parts and the whole.

This movement suggested in Luis's drawing is also encountered in the drawings made by PROEM's students. However, the movement suggested in their drawings are those present in everyday life; for instance, the movement of a car, the movement of a ferris wheel, the movement of a child playing ball, or the movement of the birds. Yet, what is observable in these drawings is that unlike the secondary students who were more concerned with internal space, the PROEM's students favoured the details of external elements. This difference may be illustrated by drawings made by secondary student Carlos, and Gil, a PROEM's student.

As we can observe (see next page) Carlos drew a kitchen enclosed within a cube that limits and encompasses the elements.

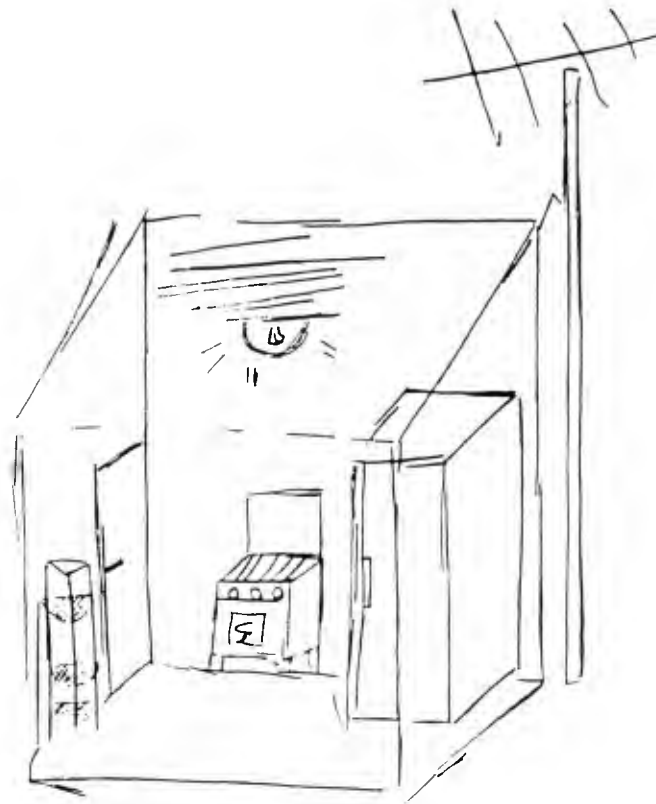


Figure 8.6 A static view

Looking at the above drawing, we can see that the elements contained are a stove, a refrigerator, a basket, a door and light on the roof. By examining the organisation of such elements it is noticeable that Carlos, as Elisa, did not consider the visual relationship of the one object relative to another since there appears to be no evident grouping of objects. Instead, all the objects are located against the wall and settled in a static position without any relationship to one another. Besides, there is nothing personal in view nor any details which would signal an individual human presence. There are none windows, and the light is on suggesting something dark and enclosed space. The only suggested linking between the interior and exterior is the TV antenna standing on the 'box'. In contrast, as the figure below shows, the student Gil showed what occurred in the outside world.

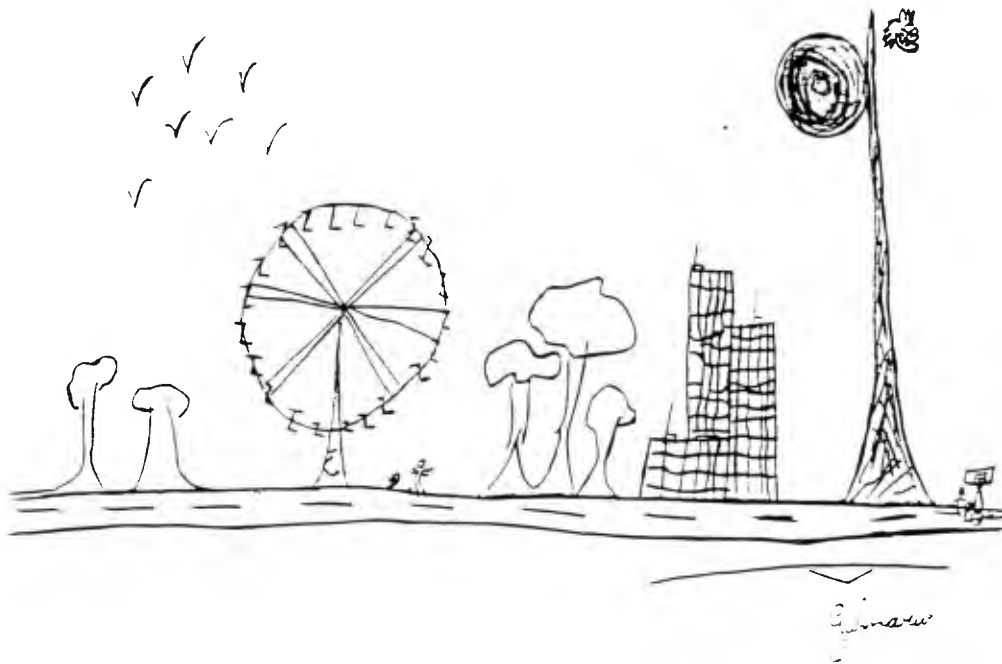


Figure 8.7 The movement of everyday life

Analysing the different drawings made by different persons, it is possible to say that in both verbal and non-verbal forms, we find a world of images. These images, in my view, represent ideas, concepts, and social beliefs.

Indeed, of considerable interest here is that if we compare the way different groups depicted geometrical forms/shapes through their free drawings with their verbal interpretations about the shapes and forms discussed in different settings we can recognise similarities between them. For instance, the

secondary students tended to relate, describe and draw geometrical shapes and forms in a static rather than dynamic way, they also tended to polarise rather than connect them; and to stress symmetry rather than asymmetry. Besides, they emphasised through their verbal interpretations and through their drawings the formal aspect of the geometrical form. In other words, the drawings made by secondary students are considered visual signs which convey a formal knowledge of the geometrical objects.

Unlike the secondary students, the manual workers' drawings were not based on symmetry but on asymmetry. Thus, instead of presenting the planes and volumes entirely separated and distinguished from each other, there was an overlapping or intersecting of elements. Besides, the elements presented in some drawings transmitted motion rather than static mode of depiction. Analysing their geometrical understanding through the visual signs constructed by manual workers, I would say that they conceive geometry as a means of describing and modelling the physical world.

In the case of PROEM's students, we can see that what was emphasised was the 'geometric world' encountered in everyday life from where those students have come to appreciate the concepts of space and geometry.

Analysing these multiplicity of visual discourses represented by their drawings, we might say that it represents specific ways of conceptualising the geometrical world. From the producer's point of view, according to Kress, it represents his or her particular 'interest' in the geometrical object, which reflects his or her place in the world.

9 Four contexts viewed by three persons

Andrea was expelled from the formal school. She repeated the first grade for three consecutive years and the second grade twice.

Elisa was preparing to enter university. She wanted to be a mathematician or a physicist like her father.

Raymond, like millions of children in Brazil, left school when he was thirteen years old in order to provide some money for his home.

The main purpose of this chapter is to explore how the above persons who were representatives of different backgrounds and geometrical experiences interpreted a cuboidal block viewed through the four settings presented in the empirical work. In so doing, my interest is to synthesise the main characteristics of each group in different situations. Moreover, I want to observe whether similar interpretations were applied in all settings or whether particular strategies were utilised according to the settings and how their very different backgrounds and goals were major influences on their readings of visual signs.

9.1 The cuboid as a visual sign

A visual sign, from the socio-semiotical perspective, is not simply a material object but an object with meaning. In what follows, I shall analyse the relationship between a cuboidal block which represented an architectural element of Brasilia city and the meaning described by three different persons.

When I asked the student Elisa, 'how would you describe it', a cuboidal block, she replied: "I see that this block has rectangular surfaces. It is obviously a solid body, therefore it has weight, breadth and height. In addition, I can see that it has six surfaces, twelve edges and right angles".

What is explicit in this brief comment is that Elisa tended to consider the block as a structure external to itself, consisting solely of width, length and

height. In doing so, she did not point to its walls, its colour and contents, i.e; as a place where people move and execute different activities. Moreover, by analysing Elisa's comment, I noted that she made no distinction between the blocks. For instance, she did not observe contrasts in their height or width (e.g. in her description there was not any comparison which involved qualities such as 'like', 'higher', or 'wider') nor did she notice any specific similarities (e.g. about similarity of their colour or details among the blocks).

Analysing Elisa's answer from a socio-semiotical perspective, I would say that the relation between the signifier (a cuboidal block) and the signified (i.e; 'it is a rectangular solid which has six surfaces, twelve edges and right angles') was not motivated or derived from the object itself. Rather, the sign produced by Elisa expressed ideas which echoed the language of formal schooling. In other words, her ideas 'reflected' a social environment where shapes and forms are conceived and constructed as a result of an axiomatic system, independent of any actual reality (like colour, texture, weight etc.). This formal discourse learned at school certainly had a considerable effect upon her visualisation and speech, although people such as Elisa may be largely unaware of it.

The logico-analytical perspective would provide us with another way of looking at Elisa's answer: Specifically, her answer appears to reflect a propensity to abstract objects and situations, of interpreting a cuboidal block without any support based on the perceptually given - cognitively speaking. Elisa produced an interpretation which was not limited by the reference to concrete perceptual values but was based on her mental representation of the object.

Let us observe what the carpenter Raymond had to say about the cuboidal block. Looking at the block, he firstly remarked: "this block is comparable to a box in that it has a top, a bottom and four sides. The sides are not equal, yet two of them are smaller than the other four". He went on:

The block was constructed based on a straight style in which the form of 'tablet' is the most important aspect of it. The block is high, higher than others. It must have twenty or thirty levels. This block is similar to that one because both are squares. The difference between them is that one has a beautiful

ornament in front of it, while the other does not. It has a long facade made of glass, and the short windowless sides are scattered with ceramic tiles.

If we analyse the above comments from the logico-analytical perspective, we might say that Raymond was incapable of withdrawal and detachment from the concrete. He focused on generic features which were inherent in fact in the block to which he was exposed, but which, in terms of the logico-analytical approach, constituted a perceptual rather than a conceptual thinking. Thus, the strategy used by Raymond is considered to be of a lower level of abstraction than that employed by Elisa.

It might well be true that when Raymond looked at the block he described a set of perceptions, but this set of perceptions in the viewing gaze cannot itself provide criteria of recognition. In other words, after analysing Raymond's observation we cannot say that the sign constructed by him was wholly alienated from its 'referent' but rather he also recognised various features of the physical environment surrounding him. Thus, his ability to recognise an object was defined in terms of a certain set of codes of recognition, codes that listed certain features of the objects which did denote a physical object but rather a cultural unity¹. If we accept this assertion, it is possible to understand why Elisa conceived the signifier 'cuboidal block' as a mathematical object, with no link to the actual reality, while Raymond, who did not have access to the formalized axiomatization of the geometrical objects, emphasised the block as an element of his environment.

In a similar way, Andrea, who also had little experience of a formal mathematical discourse, tended to describe the cuboidal block by emphasising its appearance (its exterior facade, its colour, its windows) and its function. It is significant to observe that when she described the block's appearance it did not possess any of the properties of the block presented in the model. Thus, there was no similarity, or resemblance, between the signifier and the signified. In other

¹ The critical difference between the terminology 'perception' and the terminology 'recognition', as Bryson (1991:65) reminds us, is that the latter is social.

words, the block presented in the model was constructed from white cardboard paper without any details or colour (cf. page 70). However, Andrea said: "I see many windows, curtains and doors where the politicians enter and exit. Inside of this block there are public servants who work for the government. Its colour is green, a pale green that reminds me of some houses in my home town".

Andrea's way of constructing her visualization may be interpreted in two different ways. If we analyse her construction as an explication of the relationship between 'form and perception', we would say that the information extracted was not based upon what was seen - the external surface of the block - rather, she was able to mentally visualise the 'real' block which was absent from her sight. However, the mental image constructed by her maintained an analogical character since it was based on external visual support. From a developmental perspective, she would be interpreted as not capable of separating her mental image from the physical object (the stimulus material); within this perspective, Andrea's observations would be understood as perceptual rather than theoretical concepts as her arguments did not involve a logical construction of the object or an analytical abstraction of it, as occurred with the secondary student Elisa.

But another way of analysing Andrea's visualization consists in examining not the 'relation between image and its object, but between the image and a previously culturalized content'. Thus, the concern here would be not to examine whether or not the image constructed by Andrea corresponded to the physical properties of the 'real' block, or of the block presented in the model, but rather the interest lies in examining what motivated her to construct a particular sign. Within a socio-semiotical perspective, the relationship between the signifier and the signified, constructed by Andrea was motivated by her social experience. More specifically, although the social environment experienced by her overlooked the axiomatic character of the geometrical objects - she could not recognise a cuboidal block as an abstract entity - she could find another meaning,

that the geometrical block was a 'tangible and palpable' object which was part of her cultural and physical reality.

In my view, what the above fragments demonstrate is that a visual text does not depict a single and unambiguous meaning, neither are we free to determine the relationships that we construct and perceive between the objects and our concepts about them. The meaning of an object is not fixed, nor is it arbitrary, rather it is the result of our inclusion in a certain grouping and our interaction within certain social activities that provide us with the cultural repertoire of resources which we subsequently employ.

This way of examining how people describe and interpret three-dimensionality fundamentally contrasts with the psychological approach. Empirical research on the cognitive perspective has typically been concerned with specific intellectual skills and with general mental frameworks. In examining the observer's ability to recognise the objects, events and situations depicted in images, researchers have focused on the perceptual and representational aspects by which observers structure events, and from which they derive ideas and therefore ways of describing the world. The approach taken in the present research rejects the external 'referent' (the real and actual object to which the sign can refer) as the criteria for analysing the meaning of visual text, and has shown that the relation between an image and its object is based on criteria of similarity, such as analogy and shared properties.

9.2 The production of a visual sign: wooden blocks

In analysing how the students Andrea and Elisa, and the carpenter Raymond interpreted a cuboidal block presented in the wooden blocks setting, I shall consider two different views about the production of visual sign: the first, assumes that the relationship between the signifier and signified is mediated by three elements: 'action-image-thought'. The second, considers that the production of the sign is mediated by a triadic relation between 'world-object-action'. In

doing so, I want to reinforce some points of view discussed in the previous section.

In order to examine these two ways of addressing a visual sign, I shall begin by looking at how the student Andrea described a set of wooden cuboids.

I see different sofa bases. I think that a sofa without it is worthless. So, we should use these blocks as a sofa's base. They would suffice for this purpose because the sofa would stay firm on the floor. However, the round blocks would roll and, therefore, would not work. In reality, I would like to take some of these square blocks to my house because our sofa is broken and we do not have the money to buy a new one.

From this brief construction, it is possible to examine how a physical object is converted into a visual sign. What is important here is the relationship between the signified (Andrea's idea of 'sofa bases') and the signifier (a wooden cuboid). For Andrea, the producer of the sign, the geometrical block was primarily understood as having a function; 'the block could be used in the sofa of her house'. This way of understanding a geometrical block illustrates, from the socio-semiotical perspective, a number of fundamental characteristics of the visual sign.

When one person describes an object, he/she assumes an active position with respect to what affects or interests him or her. This interest is itself a reflection of his/her position in a social life which organises the way of seeing and conceptualising the object, or the situation being communicated. Therefore, if the production of the sign is conditioned by a social environment, where people act and develop, the semiotic material is not localised in the physical form of the visual sign to which a concept refers. What is implicit in this perspective is that the meaning produced by the encounter of visual text and viewer cannot be understood as 'perceptual apprehension' - the meaning is extracted from the social context, the intrinsic social nature of the sign. In this sense, Andrea found a visual link which was related to a social reality marked by an inequality in incomes and extreme low purchasing power of money.

Let us observe how Elisa, whose socio-economical and educational background was radically different from Andrea, conceptualised the cuboidal

block. Taking the involvement in the educational discourse as a variable, I suspected that Elisa as a producer of a sign, would view and construct a geometrical block according to the codes of the formal educational system. In fact, this expectation was confirmed by my results. In contrast to the 'tangible and palpable' way expressed through Andrea's voice, Elisa emphasised the formal qualities of the block. Thus, the cuboid was characterized as having, like the cube, eight rectangular solid angles and twelve edges, equal and parallel in fours. That is, there are four side edges perpendicular to a plane, and also a second plane which is parallel to the first one. She stated that "if one of the side-edges is perpendicular to one of the bases, then all the side-edges are perpendicular to both bases ... this block is a right prism and all the side faces of a right prism are congruent rectangles". Expressing the block as a mathematical 'reality', Elisa did not make any mention of qualities such as the type of wood, texture or heaviness.

If Elisa's production of the sign were to be analysed by means of the relationship 'action-image-thought', the point of identification between the signifier and the signified would not be the discourse of formal school but rather it her 'inner mental activity' which dynamically operated upon the external object. In this case, the concepts constructed by Elisa of the signifier, 'cuboidal block', represented a mental construct. More specifically, as she was able to illustrate a geometrical block without any support from the stimulus material, the concepts constructed by her would be interpreted as the result of a process of reflective abstraction. By this process, the mind - as the basis of reflection and abstraction - played a fundamental role in the process since it is the mind that acts upon the perceived environment and adapts to it by a series of 'assimilations' and 'accommodations'.

This perspective is in contrast to the triadic relationship between 'world-object-action' where the main concern is not to conceive visualisation as a cognitive activity but rather as a social activity. Thus, the sign visualised and

constructed by Elisa is simply one directed by her formal schooling, which guided and in fact limited her action and thought.

By insisting that the way people interpret objects or situations must be examined within their socio-cultural context, I am not suggesting that people's thought and action are directly defined in terms of their social class. On the contrary, a closer inspection of my data leads us in another direction. For example, although Raymond and Andrea had the same basic working-class background, there were significant differences between their ways of interpreting a geometrical block. That is to say, it is not simply being working class that creates a different interpretation of a geometrical block, it is the articulation of that class position through discourse (in this case the discourse of a formal and non-formal schooling) which leads the interpretation in a particular direction.

Let us observe how the practical experience of the carpenter Raymond interfered in his production of the sign. As the following extract indicates, his interest did not lie in the formal aspects of the block, instead he was primarily attracted by texture and the quality of the wood. Yet, having not had access to the codes of formal education, the carpenter Raymond was not familiar with geometric terminology - "I don't know how to define this piece ... or better, I don't have any theoretical definitions for it. All I know it that it is a solid, wooden block made from a soft wood, maybe a cedar wood". Although, by emphasising the texture and the type of wood, Raymond conceded room for the spatiality of the block - "As it is a solid it occupies an amount of space, not only on top of the table [the surface] but also beyond the table, occupying an amount of space in the air since it has length, breadth and thickness". Comparing this description with Elisa's, I would say that Raymond materialised the object in a real space, while Elisa intellectualised the wooden block.

In analysing the directions taken by Elisa, Raymond and Andrea throughout their discussion on the wooden cuboid, it is clear that Elisa explicitly

concentrated her selection on formal qualities even when she was referring to a model of Brasilia, she did not value attributes such as quality of colour, heaviness and texture. Instead her connections were closely bound to geometrical properties. Raymond, by contrast, oscillated between the appearance (mainly the texture and the type of wood) and the spatiality of the block. Andrea, on the other hand, centred her observations on the functionality of the block, selected from her everyday life. The formal aspect was completely absent in the responses of both Raymond and Andrea.

9.3 The mobility of the visual sign: a photographic image

Comparing the blocks depicted in the photo with those viewed on the table, Andrea observed: "they are the same but they are presented in different ways, in different images ...". At that moment, I replied: 'But is there anything different between these blocks in the photograph and the wooden blocks? If there is, what is it?'. In answering these questions, Andrea made a distinction between 'direct observation' and a photographic image. According to Andrea, although 'they are the same', there were conflicting factors between the two 'versions' - the 'real' being more open to the sight than the image. This distinction was expressed in the following way: "I don't know how to explain it but it seems to me that when we see objects in the real world it is simpler because we can move them. Thus, we can see all the walls". She went on: "for example when you showed me that wooden block last week, I did not have any doubt that it had five walls [my emphasis]. But, looking at the same block in this beautiful photo I can only see two walls" [my emphasis]. She concluded by saying: "although they are the same they look different. In this case the block in the photo is more difficult to understand than the real block".

Responses like those given above provoke a number of reflections about how meaning is generated and conveyed in visual texts. The most meaningful is that a visual sign is not only, as Andrea's observation indicates, a fixed semiotic

entity. Rather, the semantic values vary according to coding rules defined by the context. Taking for instance, the signifier 'cuboidal block' interpreted by Andrea we can say that in one situation she noticed that the block had "five walls" and in another she saw the 'same' with "two walls"; forgetting that in earlier context she saw the block as a "sofa base". Here, we are faced with three sign-functions which according to Eco might be called ('cuboidal block'=X), ('cuboidal block'=Y) and ('cuboidal block'=K) which imply, from the socio-semiotical perspective, that the signifier 'cuboidal' is mobile and flexible. In other words, 'a visual text cannot incorporate an absolute unequivocal meaning, that is, the signifier is never co-present with a signified'.

Nevertheless, there is an important aspect in these three sign-functions constructed by Andrea which deserve our attention. It is valid to say that the contextual variable produced different interpretations (in the case of the photograph it may be linked to the blocks position in relation to the camera angle and the use of lighting). But, the limits of these interpretations are determined not by the context itself but rather by the language/codes which are not fundamentally changed by differences of situation. Thus, when Andrea expressed 'I see five or two walls in this beautiful photo' it indicated that she did not possess a formal geometrical language. Rather, her language about a geometrical object was derived from everyday life together with her own subjective reference ('it is beautiful').

Elisa, like Andrea, also made distinctions between the wooden blocks viewed through the photograph and those viewed on the table. However, her focal point was different. For Elisa, the lighting effect on the blocks produced certain ambiguities and certain "distortions of shape". Her comment on a cuboidal block placed on the right top [cf. page 71], illustrates that ambiguity: "it is not possible to affirm the block as a regular solid because there is a shadow blocking direct rays of light into its lateral surface. Thus, since its lateral is out of sight I cannot assert, with precision, if its base is a parallelogram or not". In fact, she

said "the only real possibility for calling it a regular solid is when we relate the block to the others present in the photo". Therefore, Elisa commented that the photograph "does not present the object as it really is". As she said: "the surfaces of a block, when projected on a flat paper as is the case with this photo, are not shown realistically because there are distortions of shape".

Comparing the way Elisa described the photographed cuboidal block with those viewed on the table, it is possible to state that she constructed two different sign-functions. In one context, the signifier 'cuboidal block' (called by Eco the 'sign-vehicle') was interpreted as being a "prism in which its base is rectangular". According to Elisa 'it was obvious'. In another context, we can observe that the photographed block was not so obvious and exact. In my view, what this difference demonstrates is that signifieds change and move whenever new signifiers or sign-vehicles are produced. In this specific case, we can say that instead of isolating the block as before, Elisa's tendency in the new setting was to contrast, as the above comment indicated.

Comparing the blocks depicted in the photograph with those viewed on the table, the carpenter Raymond mentioned two sets of differences: first, the absence of a horizontal baseline on the photograph meant there was no frame of reference from which to judge the position of the objects. Second, the lighting effected the meaning of the blocks. In relation to the first difference, he observed that the lack of frame of reference signified that the blocks were suspended and slanted. As he commented: "Looking at these blocks in the photo, I have the impression that they are hanging as if they were in a vacuum, since we can't see the board where the blocks are placed". Second, commenting about the effect of lighting on the blocks, Raymond observed that the distribution of light and shade reflected on the blocks "confused our vision about the blocks". For instance, "looking at the block placed in the right top [cf. page 71] it is possible to see clearly its frontal facade, while its lateral is not so clear. It's like it was hidden in fog". He also commented that: "I don't know, but it seems to me that the objects in a real world receive different

light from that shown in the photo. For example, if I were holding a real block I could see more clearly the quality and details of the wood while in this photo it is more difficult to observe them".

Analysing three sign-functions constructed by each of the above persons in the present setting ('cuboidal block'="it has two walls"); ('cuboidal block'="I cannot assert, with precision, if its base is a parallelogram or not"); and ('cuboidal block'="its lateral is not so clear. It is like it was hidden in fog"), it is possible to conclude that meaning varies with the situation and cannot be understood outside the context and that meaning does not arise from the physical world. Rather, as the above sign-functions indicate, meaning is a cultural expression which represents an understanding derived from formal and non-formal schooling.

9.4 A dynamic system of meaning: the computer screen

The main purpose of this section is to analyse how meaning is generated and conveyed by reference to the cuboidal blocks viewed through the computer screen. To situate the production and to contextualise Elisa, Raymond and Andrea's acts of perception and interpretation, it is necessary to go back to a very basic principle adopted in this thesis: that geometric figures depicted on a screen are not simply formal entities contained in an arbitrary closed system upon which the subject acts in order to develop meaning. Rather, figures depicted on the computer are understood as social elements. Within this perspective, the spatial properties and conceptual qualities of the geometrical figures described by Elisa, Raymond and Andrea will be analysed from two sources: their social knowledge about the figures and their articulation with the computer where, the social subject is invited to take up an active position.

In what follows, I shall examine how this articulation took place while Elisa, Raymond and Andrea were acting upon the cuboidal block on a screen. In

this activity², a set of three-dimensional figures was displayed on the screen, where Elisa, Raymond and Andrea had the chance to work with and transform the cuboidal block. More specifically, they had the opportunity to create, rotate, move, and enlarge the figures, not available in other activities on the computer and settings of my study. Thus, as a secondary student observed; "since there is the possibility of moving and enlarging these figures I may say that we can make innumerable combinations". Elisa agreed with her classmate's point of view: "playing with these figures, as I'm doing now, I may say that we can build various compositions with those figures".

Indeed, she moved and created various figures during the present activity.

While she was doing this she observed:

Moving these figures as I'm doing right now reminds me that geometry is a composition of planes of which you assemble and separate those planes, right? Thus, if you know how to make an analysis of figures, you decompose them and make the analysis of a set composed by planes. For example, let me see ... using the computer we can move the $n^{\circ}10$ [cf. figure 3.8 page 75) to a frontal view and stick a circle in it. We see a cylinder inside a cuboid [see below]. In linking these two figures, in my view, we are analysing it.

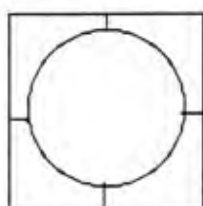
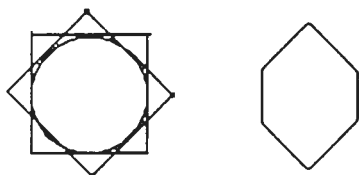


Figure 9.1 Linking a cylinder with a cuboid

At that moment, I asked Elisa what she meant by 'analysing'. She explained it giving the following example:

Let me explain it through this drawing that I have just made [see next page]. Suppose that I pass different lines at 45 degrees in this circle and square. I mean, the four corners of that square can be cut to make another figure ... this figure will be a hexagon. Thus, we can change one shape into another by alliterating the corners; in doing this we may analyse the figure, right?.

² As was mentioned in Chapter 3, in order to organise the data, I divided the settings shown on the computer into four parts, called activities 1 to 4. The activity analysed in this section corresponds to the activity 4 [cf. page 79].



9.2 Decomposing a figure

Examining the verbal and the visual utterances constructed by Elisa we notice that she produced several types of connections, movements and transformations in the geometrical figures: she connected the cuboidal block with the cylindrical, she changed the shapes, she created new shapes and she changed spatial relationships.

In order to analyse these multiple sets of relationships constructed by Elisa, I shall start by looking at how the signifier 'cuboidal block' was conceived while she was acting upon a set of three-dimensional figures displayed on the screen when compared with other conceptions made in the other activities on the computer and other settings. In my view, there are at least two main differences: First, and the most evident, is that while in other contexts she strongly tended to polarise the objects, in this activity Elisa clearly connected them. Second, instead of seeing and conceiving the cuboidal block as a whole and static unity as before, she decomposed the whole given figure into parts and combined these parts in another figure. Another important difference pointed out by Elisa in the present activity was her comment about geometry which differed from other situations present in my study, i.e; when I asked her two direct questions about geometry: 'What is geometry'? and 'What do you know about it?'. The positions established during these two situations seems to be quite different. Briefly, we may say that while was working on the screen Elisa presented a 'dynamic' position as the following comment illustrates: "Moving these figures as I'm doing right now reminds me that geometry is a composition of planes of which you assemble and separate those planes, right?"; while in the other context she characterised geometry as a set of rules to apply (cf. chapter 5 page 103).

If we analyse these differences from a socio-semiotical perspective, we might say two things: first, observing Elisa's way of constructing the relationships between the figures, we may say that the figures were connected through formal operations. More specifically, the relationship between signifier and signified was established with exclusive attention to the formal designs or structural properties of figures. This, represents a specific way of conceptualising geometrical objects, the one of formal schooling with which Elisa was involved. Second, the signs constructed by Elisa cannot be divorced from the circumstances in which they were produced. The connections, movements and transformations produced by Elisa on the set of three-dimensional figures displayed on the screen can be attributed to the possibility of exploring shapes and forms in a manner more dynamic than in other activities and settings thus motivating new means of expression to interpret the geometrical objects. This demonstrates the mutable character of the visual sign: the potential to be produced differently in different moments.

Let us observe how the carpenter Raymond interacted with the cuboidal block present in this activity. The first observation made by him was: "all of these figures seem as if they were made using a wire or they were made by transparent glass". When playing with the figures, he commented:

Right now, playing with those figures on the computer, I'm reminded of an example that I gave to you a couple of days ago. I mean, when I explained to you the possibility to carve a rectangle [cuboid] by cutting away the wood until forming a pin [cylinder]. Well, now moving these figures I see something like that. I mean, moving those figures seems that it is possible to carve the objects like those wooden blocks but in a different way. I mean these figures seem empty, different thus from a solid block. In the wooden blocks we have to use a knife to cut away the wood, right? Here on the computer it seems that if we want to carve the figures we have to draw and move the figures thus we may separate and link different parts of it. In doing this, we can see the external part of the figure as well the inside one as if it were carved.

In attempting to illustrate his comprehension, Raymond created two sets of movements on the screen: first, he reduced and moved a loaded circle [see figure 9.3 below] to the top of the nº 10 fig. 3.8 [the "transparent" cuboidal block]. In doing so, he expressed: "as I said before, it is possible to make a "tube" [cylinder]

from a rectangular block [a cuboidal block]. Also, it is possible to make another figure from that "tube". After saying this, he produced two drawings [see figure 9.4 below].



Figure 9.3 Connecting a cuboid with a cylinder

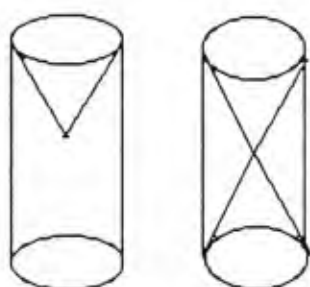


Figure 9.4 Dividing a figure into cross-sections

Looking at his drawings, and reading the above comments made by him, we may say that there were some similarities between the way the carpenter Raymond and the student Elisa interacted with the geometrical figures at the present activity. That is, both "played" with the figures, both connected the cuboidal block with the cylindrical, both created, dislocated and overlapped figures and if we compare the visual and spatial qualities of the figures constructed by Raymond (mainly his second drawings) with those produced by Elisa, we may say that there was at least one similarity between them: the elements were organised symmetrically.

However, there was a crucial difference between them. That is, the relationship between the signifier and the signified constructed by Raymond and Elisa was established in a diametrical way. While Elisa established a relation based on a logical quality with exclusive attention to the formal designs or structural properties of figures, Raymond made a connection based on the 'materiality' of the figures. In this view, he did not relate an 'ideal' cuboidal block with a cylinder or a cylinder with one or two conical sections with which had no colour, no material substance, no mass. Rather, he connected those geometrical figures in order to compose a concrete object. A geometrical object or a geometrical piece of his everyday life: an "old fashioned filter made of cloth" and an "hour glass". According to Raymond, "they look nice on the screen" [see figure 9.4 above].

If we are reading this differentiation in Bakhtinian terms, we might say that it represents specific ways of conceptualising the geometrical objects, which are effects of the practices with which the people are involved. More specifically, Elisa constructed a sign which 'reflected' her actual social context in which a geometrical block was conceptualised as a mental entity ('the so-called geometrical figures') while for Raymond, geometrical objects were neither ideal nor abstract. Instead, they were physical events which he moulds, handles and shapes.

Within this perspective, it is possible to understand why Andrea, a PROEM's student, did not make any construction or decomposition on the screen neither had much to say about the form of the cuboidal block but she moved the figures displayed on the screen saying that: "this movement is like to dance lambada" .

The main purpose of this chapter was to demonstrate how I have tried to rethink and conceptualise geometrical awareness which is not based on the traditional psychological underpinnings. The starting point of my attempt was to examine elementary geometrical knowledge and concepts not as an abstract

system of codes and rules which are taken to be universal regardless of historical time and place. Instead, my interest was to situate its categories within specific cultural and social context. This way of proceeding had important methodological consequences. Rather than concentrating on axiomatic aspects of the canonical 3-D forms/shapes, I wanted to examine how 3-D geometrical elements - as cultural artefacts - were linked to the participant's everyday life scenario.

In order to analyse 3-D forms/shapes as 'concrete expressions', I explored through the interviews what kind of interconnections the different groups made when relating their everyday life situations with the different settings displayed. Analysing the way Elisa, Raymond and Andrea, who were representatives of different backgrounds and geometrical experiences, interpreted a cuboidal block viewed through the four settings presented in the empirical work, we can say that there were contrasts in their way of relating geometrical forms and shapes. Looking at these differences from the theoretical framework adopted in this research, I would argue that these divergences were not rooted in personal choices - as some logico-analytical studies claimed - but rather these differences were effects of the specific practices into which the persons were inserted. As soon as it is proposed that differences in the social forms of activity affected the way people conceived geometrical elements, it has then to be claimed that interpretations were various, a function of various activities and practices which constitute social life.

Moreover, my empirical data shows clearly how a range of representations can produce differences of meaning, that is, how one signifier - in our case, geometrical material or pictorial figures - can, given the context, denote various signifieds. What clearly underlines this assumption is that signs change as the situation and context change.

10 Conclusions

This thesis has offered a re-vision of the logical-mathematical approach to the study of geometrical understanding. The concept of re-vision, as the etymology of the word makes plain, means 'the act of looking back, of seeing with fresh eyes, of approaching an old text from a new critical perspective'¹.

My task involved a series of shifts in its principal focus of interest, moving from the Piagetian perspective which conceives geometrical knowledge as a cognitive process, through a set of concerns around the construction of mental images as a product of interiorization of intellectual acts, towards a perspective which recognises that geometrical knowledge is rooted in the material realities of people's lives.

In choosing this way of conceiving geometrical awareness, I have sought to open up new connections that might offer, to some extent, theoretical contributions and insights into the understanding of geometrical awareness. The suggestion is still of a tentative and exploratory character. Firstly, there is still a great deal of uncertainty about the precise implications of this alternative approach for the field of geometry education since its theoretical foundation and the analytical concepts used of 'sign', 'sign-functions', 'visual text', and 'heteroglossia' have not been used by previous research in this way. Their meanings would need to be probed with more precision through further investigation. Secondly, the strategies designed to distinguish different types and levels of geometrical discourse have been established within a theoretical framework whose main interest is to search out how social experience influences the way of interpreting situations or objects.

Although my attempt to offer an alternative approach is limited to a specific proposal and still needs to be deepened and extended, the attempt to establish strategies of analysis that take into account the social and cultural

¹ Adriene Rich, *On Lies, Secrets, and Silence*. 1979. pg. 35

dimension of geometrical understanding requires one to call into question some issues which have not to date been adequately investigated and may have important implications for geometry education.

10.1 The traditional model

Mathematics researchers are aware of the fact that cultural and social experience affects ways of constructing and thinking mathematically. However, in the case of geometry education, the message of this thesis is that researchers have not sufficiently reflected on how far geometry is part of a cultural and social situation. Rather, they tend to concentrate on a micro-level of analysis, that is, on the internal dynamics of situated activity (settings or contexts) without questioning how the macro - the larger social context - is implicated in those activities².

The exclusive focus on the micro-level, or on what I see as an inadequate 'dialogue' between these two levels of analysis is, I believe, one of the most important problems that the current research on geometry has to face. In my view, these two levels cannot be understood as two types of reflection which should be strictly separated as if there were an irreconcilable conflict and opposition between them. Rather, I would say that each level of analysis reveals different aspects of a complex but unified phenomenon.

Thus, small fragments constructed and expressed through the empirical work should not be understood as a product of a subjective act or a result of a logical and mental interaction between the individual and the objects. If we admit that there is an interaction between micro and macro levels we cannot produce empirical analysis overlooking the fact that what the individual creates in a micro-interaction is always a social gesture, a set of social values and beliefs.

² The concept of the macro-micro problem is adopted from the works of Layder (1993) and Coll and Onrubia (1994).

Another important issue which has been overlooked by much current research and which this thesis has sought to address has to do with the need to account for the variety of geometrical experiences of diverse groups. Although in other areas of mathematics this differentiation has been recognised, as reported in Chapter 2, in geometry the tendency has been to view knowledge within a universalistic framework; that is, to bind together different forms of geometrical diversity. Within this perspective, there is an assumption that all forms of diversity may be understood from the same basis of a set of universal elements such as 'human being', 'class' or 'race'. In so doing, current research on geometry often deals in broad generalisations, treating a wide of range of individual knowledge as homogeneous, undifferentiated phenomena. Above all, geometrical 'skills' are identified which are common to all cultural groups which are then ascribed to be the structure of the human mind that develops by a series of actions upon concrete objects³.

Yet, when geometry education researchers compare and contrast different individuals or groups, there is a strong inclination to categorise geometrical knowledge within a set of hierarchical relations of difference, lower versus higher. The point of attention here is to consider a student's competence in conceptualising the geometrical entity as a formal structure only. Here, the word 'competence', not only denotes the capacity to identify and decode certain conceptual properties 'intrinsically' present in the visual model, but also the capacity to integrate the 'conceptual and figural properties in a unitary mental structure'. By examining the articulations of the subject upon geometrical objects, it is noticeable that most mathematics education researchers conceptualise this relation in terms of the transmission of information. This transmission model involves a linear communication between an individual subject and an individual visual text in which the subject is an active decoder of the message. Her or his role, therefore, is to extract the semantic properties of

³ These tendencies are very evident in the works of Bishop and Mitchelmore.

the visual text; and the function of the text is to stimulate the inner images of the subject. This interaction is considered, by itself, to be the condition for a complete awareness in geometry.

From my perspective, this framework has some pedagogical problems: geometry teaching and learning is reduced to an issue of correspondence between internal and external representations, in which learners, individually or in groups, are classified according to cognitive levels. This cognitive 'conflict' should not be ignored. However, in emphasising the individual's internal mental process the researchers have little to say about issues related to social functions and interactions.

10.2 First steps to a new model

In my opinion the value and importance of the socio-semiotical theory for the study of geometry is that it challenges the dichotomy between an individual subject and an individual visual text giving possibly a fresh start to the study of geometry. For instance, Eco's notions of sign-functions and his criticisms of iconicity, Kress's notions about sign and Bakhtin/Voloshinov's notion of heteroglossia together with his understanding of sign and reality, introduce new possibilities for rethinking geometrical images as more than simply an iconic sign in which meaning is created from the various clues and evidence provided by its figural features.

If we abandon the assumption that meaning is not exclusively derived from the visual medium, we are rejecting this formalist option - one which limits the production of meaning to the immanent logic of the object itself - as an internal organisation or as inherent essence. We are also discarding the universality of the geometrical process since concepts and objects do not relate as mind to matter. By dispersing abstract generalisations, some categories such as 'mental' or 'internal' undergo an explicit devaluation. In turn, geometry is conceived as a product of different social practices that lead to different forms of constructing

geometry and different ways of thinking geometrically. If this assumption is accepted, the widely-held belief of transparency between form and function, signified and signifier, structure and meaning also would be contested. In turn, conflict would be observed rather than harmony, difference rather than resemblance.

What makes this last observation suggestive for education is the opening it provides for a more complex view of meaning. Redefining the articulation between structure and meaning, signifier and signified, this perspective not only allows for examining points of conflict, discrepancies, and ambiguities among different ways of constructing and thinking geometrically which are culturally and socially produced, but also enables one to search out the contextual dynamics in which meaning is constructed. By setting a geometrical text in play and examining it in different contexts and medium, we may learn that even stable visual properties, such as geometrical figures, are polysemic rather than 'textual and fixed'.

Based on the supposition that signification is not produced in a uniform way but according to the contextual dynamics in which the meanings are constructed I developed some analytical tools like 'sign', 'sign-functions', 'visual text', and 'heteroglossia' which fundamentally stress diversity and mobility. I then designed an empirical study where possibilities for multi-interpretations could emerge and converge and sought to investigate the nature and extent of diversity.

In attempting to explore how people would interpret and describe geometrical elements, I selected three sets of people as illustrative of different socio-economical and educational backgrounds. A striking difference among the sixteen people that composed the three groups, fundamental to the present study, was their geometrical experience. More specifically, I wanted to consider differences of geometric understandings between students who were in contact with school geometry for over four years, those who had almost no formal

knowledge of geometry, and those who had contact with geometrical shapes and forms through their work experience.

In order to examine how geometrical elements would be described and interpreted by the groups considered, four settings were presented. The main purpose of the settings was to provide different sorts of contexts in which a set of three-dimensional elements and pictorial figures of different forms and sizes could be discussed. The material included: a model of Brasilia; wooden blocks with various sizes and shapes; a photograph of the wooden blocks; the model of Brasilia and a set of 3-D geometrical figures of different shapes modelled on a computer. Special attention was given to five geometrical objects: a cuboid, a cube, a cylinder, a sphere, and the hemisphere. The choice of these geometrical shapes was inspired by those that constitute the architectural design of the central area of Brasilia, called *Plaza of the Esplanade*, which has particular significance to Brazilians in general, and more specifically the participants in my study who live in Brasilia.

I distinguished the following aims for the empirical part of my study:

- (i) to map the interpretations of the various geometrical objects
- (ii) to examine how and why they vary
- (iii) to relate these variations to cultural/social factors and to the relationship between formal and practical knowledge
- (iv) to examine how different visual media may motivate some interpretations and inhibit others

The results demonstrated that there was indeed a plurality of messages produced by different participants and in the different contexts but there were distinctions and similarities between groups across contexts. The basic distinctions between the groups were: the group of secondary students had difficulty in relating forms and shapes to their everyday life situations; in contrast, the group of adolescents with almost no formal study in geometry easily made connections with their environment, while the manual workers tended to associate the geometrical forms with their work.

In interpreting the geometrical elements the fundamental differences were: the secondary students tended to describe the elements by their geometrical attributes and formal qualities. In expressing the 'visual texts' (images and objects) presented to them as a mathematical construction, they tended to overlook attributes such as quality of colour, heaviness, and texture. Yet, there was a strong tendency to polarise differences and emphasise similarities, and an absence of observation which suggested movement. In contrast, the group of adolescents with almost no formal study in geometry strongly associated the elements with familiar objects in which size and shape were not their focal point of attention. Rather, the group, on the whole, made reference to a series of attributes drawn from their everyday context such as colour, material, and ornamental details. Yet, they tended to contrast rather than to polarise the elements; emphasised functional and subjective aspects in their observations, and their interpretations frequently suggested movement. In a similar way, the group of manual workers instead of polarising the elements they contrasted them. Yet, their focal point of attention was not the geometrical features of the elements but they emphasised the type of wood, texture, colour, and heaviness. However, the aspect of movement was frequently mentioned and subjective and functional criteria were central. In contrast to other groups, the manual workers, tended to describe the elements, as if were moulding or handling them.

Although the PROEM's students and the manual workers, like the secondary students, commented on the dimensionality of the figures and the blocks, they differed in their mode of conception. That is, instead of stressing a block as a piece of matter occupying an abstract three dimensional space (a geometric body) which contrasted with the pictorial figure as an absolute two-dimensional enclosed plane, all the PROEM's students and the manual workers compared a physical body occupying a real space with an abstract plane surface. That is, while the secondary students emphasised an intellectual concept of space - a symbolic space - the other groups materialised the objects, stressing an

empirical visual experience of space. What is explicit here is the tangible way of describing a geometrical element contrasting with the generalisations and abstractions formulated by the secondary students.

The present work has sought to understand these different responses and interpretations as flowing from the participants' insertion in certain cultural and social activities - cultural clusters which guide an individual's interpretation of messages. Thus, when the participants of my research used a language for describing and interpreting 'visual texts' (images and objects) presented in the empirical work, they were not only using language to transmit a semantic referential concept but were transmitting values, practices, and experiences associated with those images and objects.

To argue that an individual's interpretation of a visual text must be seen in its social context is by no means to opt for a mode of determinist explanation in which the individual interpretation is directly explained by social position. This would imply a clear-cut opposition or a well-defined border between the groups in the research. Rather, as the results showed, there were points of conflict, divergence and ambiguities within groups and indeed similarities across groups. For example, for all three groups the signifier 'cylindrical and hemispherical blocks' were the most difficult to analyse. The reason for this difficulty did however vary across groups. The secondary students had a school-based reason. In explaining this difficulty, the secondary students commented that the sphere/hemisphere in solid geometry was the last topic to be studied in the mathematics curriculum. Due to time constraints most of solid geometry was frequently missed. The manual workers, particularly the carpenters and graphic designers, had reasons from their practice. They explained this difficulty by their familiarity with straight objects and their unfamiliarity with round ones. The group of adolescents with almost no formal study in geometry justified their difficulty in analysing the cylindrical and hemispherical blocks by reason of the unclear delimitation of the edges. What underlines these explanations is that

particular experiences produced specific readings, whose outcomes only appear similar from a superficial analysis.

In rejecting the universality of meaning, and the view of meaning as resulting from a union between signifier and signified, the theoretical concepts adopted in my research strongly suggest that sign, as a mapping of meanings, is not only read in different ways as products of background, experience, and social motivation but also as products of the context in which the meaning is constructed. What, inevitably, emerges from this assumption is mobility, variation and flexibility of meaning.

Indeed, in providing different sorts of contexts, I expected that the presence of the same shape in different media would produce distinct sorts of interpretation. This expectation was linked with contextual relationship - the semantic mobility - between a visual text and its meanings. I expected that presenting the geometrical elements in a photograph - as visual media - would contribute to the production of particular meanings about the blocks. I also suspected that the nature of the computer's message might also provoke certain readings of the forms or shapes. These expectations were supported by the results.

The photograph had an effect on the viewer which was associated with the way a camera framed the images and focused on some of its aspects by the use of lighting. Building on this, I believed that a visual text within a photograph might provoke new interpretations and concepts. In fact, there were striking differences in the way the groups interpreted the blocks depicted in the photograph and in the other contexts. That is, all commented about the absence of a horizontal baseline in the photograph which meant there was no frame of reference from which to judge the position of the objects. The lack of this frame of reference meant that all the participants mentioned that the blocks were suspended and slanted. Yet, all the groups noticed how the surfaces of the objects looked different according to their position in relation to the camera angle and the

use of lighting in the photograph; this effect of the light suggested new meanings for the blocks. For example, when looking at the blocks in the photograph, the secondary students described geometrical features and properties not mentioned in other contexts; the other groups also noticed differences in relation to their texture and appearance.

Similarly with the computer, the medium facilitated the manipulation of the blocks which provoked a shift in meaning. The dynamic mode of viewing the objects, not present in other contexts, may explain the tendency amongst the secondary students and manual workers groups to connect or to decompose the shapes or forms when using the computer. Thus, they constructed different relationships between the signifier and signified. More specifically, there were at least three main differences: first, and the most evident, is that while in non-computer contexts the secondary students tended to polarise differences, when using the computer, they connected the geometrical elements. Second, instead of seeing and conceiving the geometrical elements as a whole and static unity as before, they decomposed the whole given figure into parts and another figure. Similarly, if we examine the connections made by the manual workers within the computer context, it is possible to say that new concepts and meanings about the geometrical figures were constructed. For instance, while in other contexts, i.e; the wooden blocks, the manual workers connected or interacted with the elements as if they were moulding or shaping a solid block, in the present context they tended to make relations in which the focus was on the decomposition of the figures. However, the secondary students also noticed something odd in the way the objects were displayed on the computer screen. This "strangeness" provoked commentaries which did not appear in other contexts. These differences illustrate, from a socio-semiotical perspective, that a signifier is never co-present with a signified. Rather the semantic value varies according to coding rules given each time by context.

Nevertheless, as the results showed, meaning is not determined by the visual medium itself but its influence is intertwined with the social experiences interacting with the geometrical elements to produce together different ways of constructing and thinking geometrically.

In contextualising meaning, the research also indicated that the modes of communication and the strategies used during the process of probing geometrical understanding had important implication on the nature of the answer obtained. In fact, my empirical work illustrates that meaning changes as the type of communication changes. More specifically, the initial sessions of my empirical work involved discussions that enabled the respondents to formulate and articulate their own interpretations about the material presented to them. Analysing the interaction of the participants in this context - where there was room for spontaneity and improvisation - it is possible to say that all groups fully explored their ideas about geometry. However, in the context created in our last interview some of them became monosyllabic.

More specifically, when the secondary students answered the direct questions 'what is geometry' and 'what do you know about it' they had no difficulty in theorising or applying rules about the shapes and forms. In contrast, when the manual workers and the PROEM's students are asked to speak about geometry as abstract school knowledge - as opposed to their own practices with geometry - they do not have much to say. However, my results demonstrate that though they do not know geometry as abstract school knowledge they actually know strategies and methods which are cultural and social expressions of a lived experience with geometry.

Considering these differences, in the context of my study, I would suggest that the students enrolled in a formal school learn about geometry, the manual workers learn through geometry, whereas the students enrolled in an "alternative school" learn geometry from life.

10.3 Implications for education and for further research

Analysing the way the participants of my study conceptualised the geometrical elements in different settings presented to them during my empirical work, I want to conclude this thesis by pointing to the need for a reconsideration of some ideas that have guided the geometry education during the past decade.

First, as was mentioned in Chapter 7, in recent years there has been much research involving geometry and the computer. When mathematics education researchers examine this connection, the tendency is to enquire whether the strongly visually-based computer environment and its interactive potential facilitates the construction of geometrical representations and understandings. In this debate, the computer is usually conceptualised as an environment - a stimulus material - which can be used to provide "both virtual worlds of realities and realities materialising theoretical concepts" (Laborde 1995:261). In this way, ideal qualities of geometrical concepts are made more tangible for inspection, manipulation, and discussion, thereby encouraging "mental experiments of image-reasoning" and a "correct interaction between the figural and the conceptual components of geometrical reasoning" (Mariotti 1994 114-115). This view sees the computer environment as a place where a rational being finds a "transparent" geometrical form in order to fill it out or to find in it the ideal and logical features of the figure. In this view, a geometrical form on the screen is either a signifier having 'its own perceptual signified which is determined by figural organisation laws, and by pictures clues' or it is an abstract recipient, a 'naked mannequin waiting to be clothed' by the logical and intellectual responses made to it.

The research reported here on the way the three different groups of the study interacted with the computer casts doubt on this conception of the computer as a simple stimulus or reference. In other words, a geometrical object depicted on a screen can be seen as an object of aesthetic experience about which the

individual can exercise different judgements - not necessarily limited to the rational or intellectual considerations but acts of imagination and enjoyment that constitutes an experience - the experience of seeing a geometrical element on a screen.

If this view is accepted, we might not refer to geometrical forms as a simple structure within figures which are conceptually abstract. Rather, we might refer judgements of geometrical value to social assumptions, priorities and interests. In this, geometrical forms are not empty and skeletal, but replete and vital.

Implicit in this way of conceptualising the computer, is that meaning is not derived from the simple manipulation of the internal codes and structures of a geometrical text, neither is a perceptual apprehension conceived by an individual in isolation.

Secondly, I would like to think through the implications of diversity. It seems to me that to recognise plurality and diversity in modes of constructing and understanding geometry means to reject assembling heterogeneous possibilities into general and fixed dichotomies such as abstract/concrete, upper/lower, scientific/practice. Central to these binary distinctions is the question of the status of a discourse within society and culture in which 'abstract theory' or 'pure science' achieves "high ground, as well as high status" while 'practice' is seen "as demeaning and lacking in prestige" (Laydon 1991:48).

Rather than undermining one or another, it may be more useful to explore possible articulations and implications of different ways of constructing and thinking geometrically. In so doing, we may learn to see the virtues of different experiences and above all, we would examine an important area of enquiry which has not been debated by current research: Why learn geometry and who should learn it? What happens with geometry outside school? How is geometry actually 'formulated and elaborated' in out of school contexts and how can it be related to the geometry allegedly 'practised' in school?; How do peers discuss their

geometrical understanding? In other words, how might the use of geometry as a form of language be understood as instrumental in the acquisition of geometrical knowledge [my emphasis], and as an activity closely related to the concrete, practical experience? Thus, in the very specific contexts of practice, it is not possible to talk about geometrical cognitive elaboration without mentioning its social function.

By incorporating social dimension within the field of geometry it may encourage researchers to pursue a route where the meaning of images is neither an inward part of the mental process nor in the visual text which defines meaning in its entirety (the formalist position). If this assumption is accepted, researchers can start to explore questions like: how 'inner images' are mediated by material conditions and the contextual dynamic in and through which they are constructed; how the specific forms of curricular knowledge contribute to these images; how these images are for example affected by the way geometrical figures are represented in school textbooks; and how teacher's representations influence the structuration of the images produced by students.

APPENDIX

Secondary students	Name	Geometrical Experience
	Marcelo	He had studied geometry (including the geometry of solids) for over four years
	Alex	He had studied geometry for five years (including Descriptive geometry and geometry of solids)
	Ana	She had studied formal geometry (including the geometry of solids) for over four years
	Elisa	She had studied geometry for over six years. Four years in Brazil and two years in France where she lived for five years
	Carlos	He had studied geometry (including the geometry of solids) for over four years
PROEM's students	Kelvin	He was enrolled at fourth grade at the primary level. No formal study in geometry
	Gil	He was enrolled at fourth grade at the primary level. No formal study in geometry
	Antonio	Apprentice of carpentry. He was enrolled at fifth grade at the primary level. No formal study in geometry
	Maria	She repeated the first and second grade many times. No formal study in geometry
	Andrea	She repeated the first and second grade five times. No formal study in geometry

Manual Workers

Tom	Potter - very little experience of formal education. No formal study in geometry
David	Potter - His school experience was almost nil. No formal study in geometry
Mario	Graphic designer. He studied some geometry while he was working as a designer probationer
Luis	Graphic designer. He was registered in a rural school until the age of ten. No formal study in geometry
Sebastian	Carpenter. He received formal schooling until ten years old. No formal study in geometry
Raymond	Carpenter. He left school when he was thirteen years old. No formal study in geometry

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